

Modeling Variability

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Material

<http://mathieuacher.com/teaching/MDE/MRI1516/>

Plan

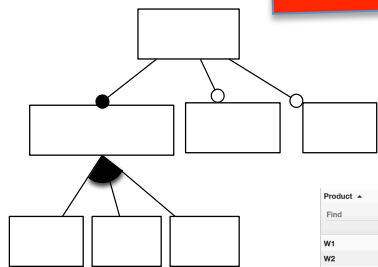
- Challenges and Overview
 - Developing billions of software product is hard but now a common practice
- Implementing Variability
 - Revisit of existing techniques and curriculum
- Specificity of Product Line Engineering
 - Process, methods
- **Feature Models**
 - **Defacto standard for modeling product lines and variability**
 - **Syntax, semantics, automated reasoning, synthesis**

Contract

- The idea of software product lines and variability
 - You will be able to recognize this class of systems
 - Aware of the complexity, the specific development process, and existing techniques
- **Feature modeling**
 - **A widely used formalism for modeling product lines and configurable systems in a broad sense**
- **Composing/Decomposing feature models with a domain-specific language**
- **Reverse engineering variability models**

Modeling Variability

Modeling and Reverse Engineering **Variability**

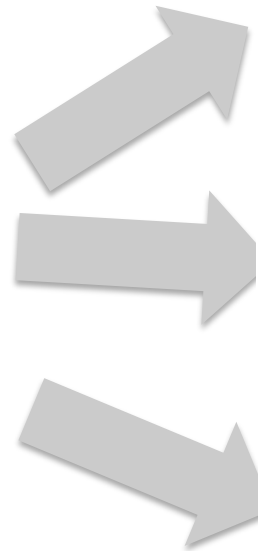


not, and, or, implies

Product	License	Price	Language Support	Language	WYSIWIG
W1	Commercial	10	Yes	Java	Yes
W2	NoLimit	20	No	Java	Yes
W3	NoLimit	10	No	Java	Yes
W4	GPL	0	Yes	Python	Yes
W5	GPL	0	Yes	Perl	Yes
W6	GPL	10	Yes	Perl	Yes
W7	GPL	0	Yes	PHP	No
W8	GPL	10	Yes	PHP	Yes

Feature models
or Product Matrices

(product lines)



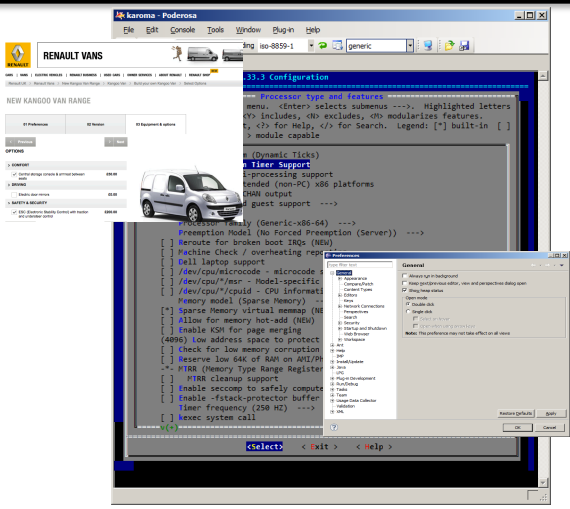
Variants of code (e.g., Java or C)
Variants of user interfaces
Variants of video sequences
Variants of models (e.g., UML or SysML)



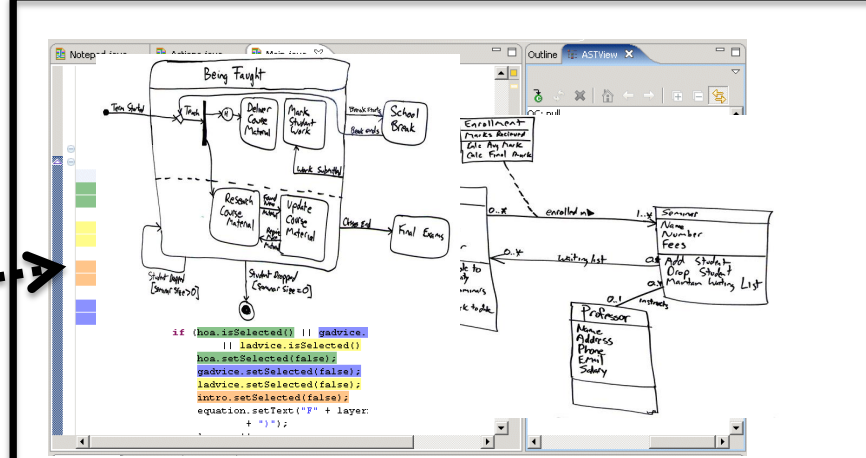
Variants of « things » (3D models)



...



Variability Model



Base Artefacts (e.g., models)

mapping

✓
✓
Configuration



Software Generator
(derivation engine)

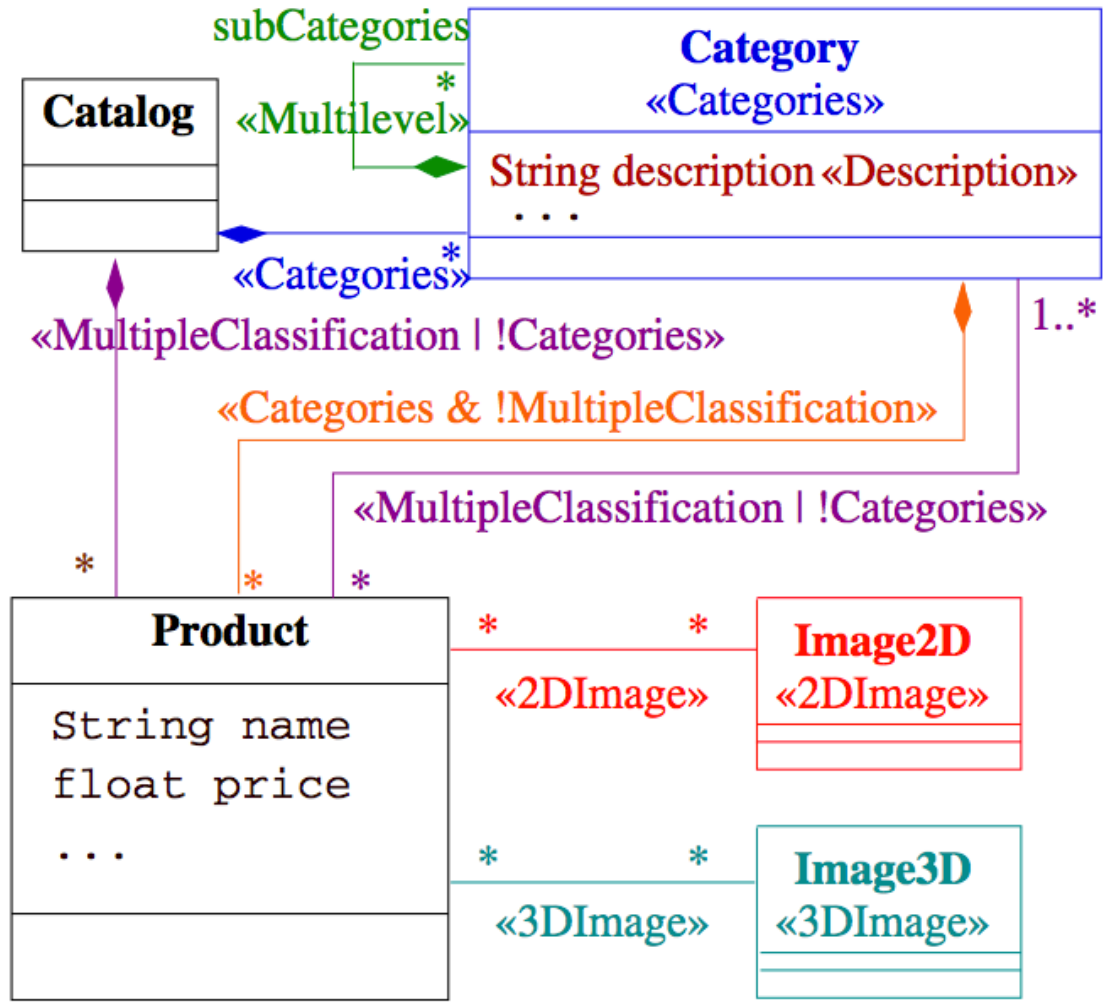
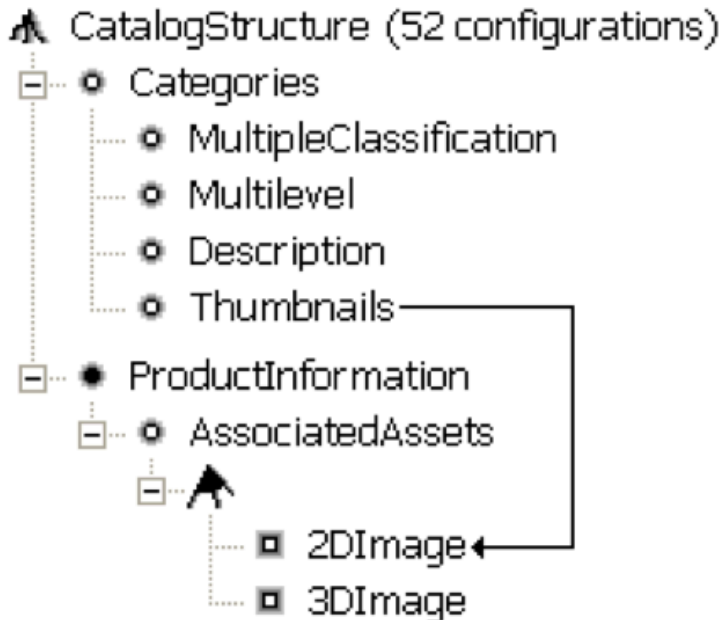


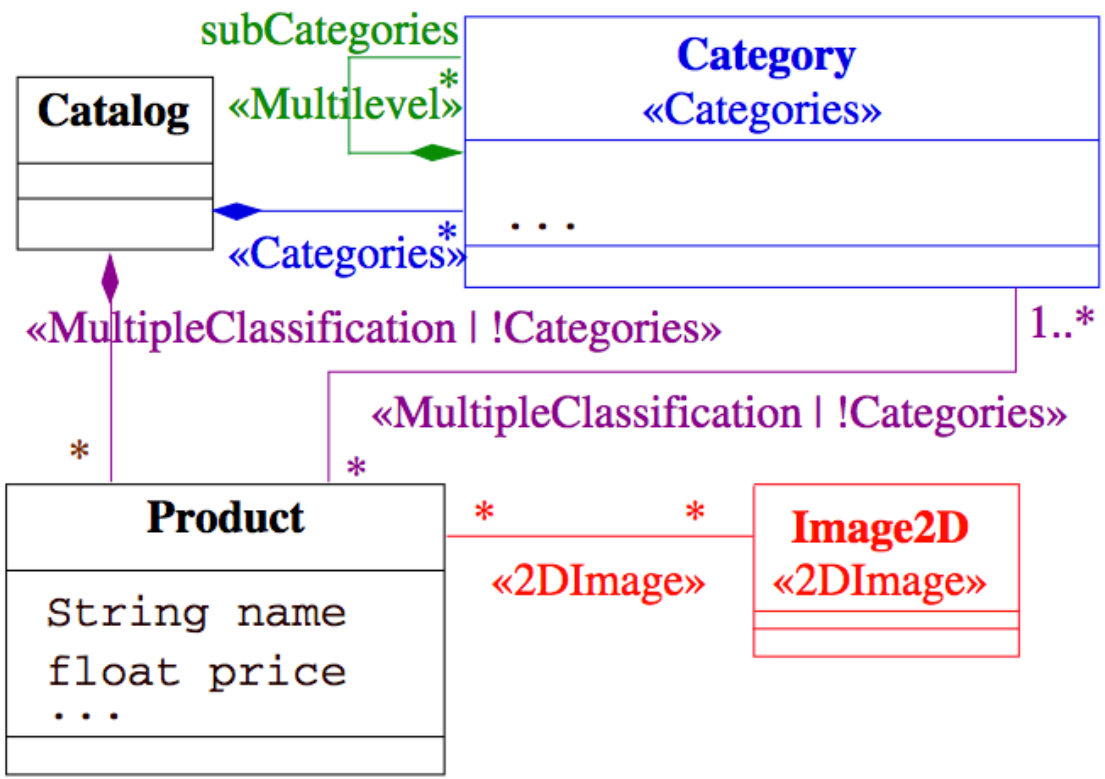
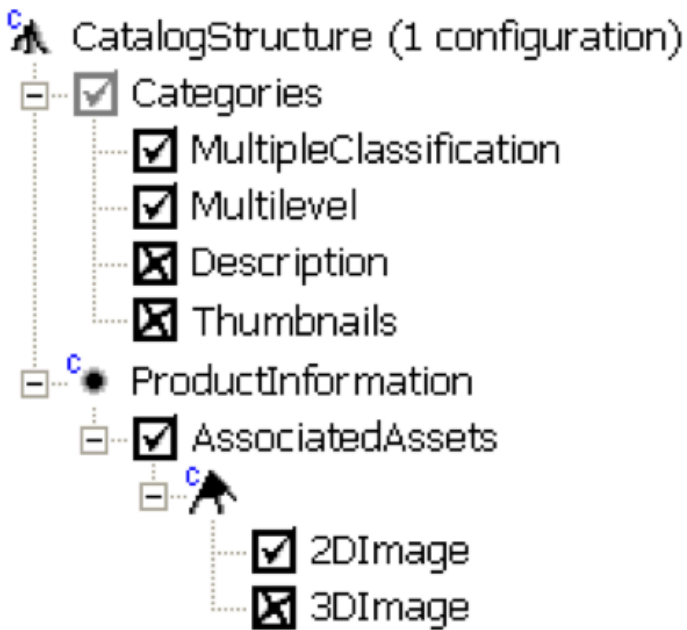
Quizz: what are the constraints over WORLD and BYE?

```
#include <stdio.h>

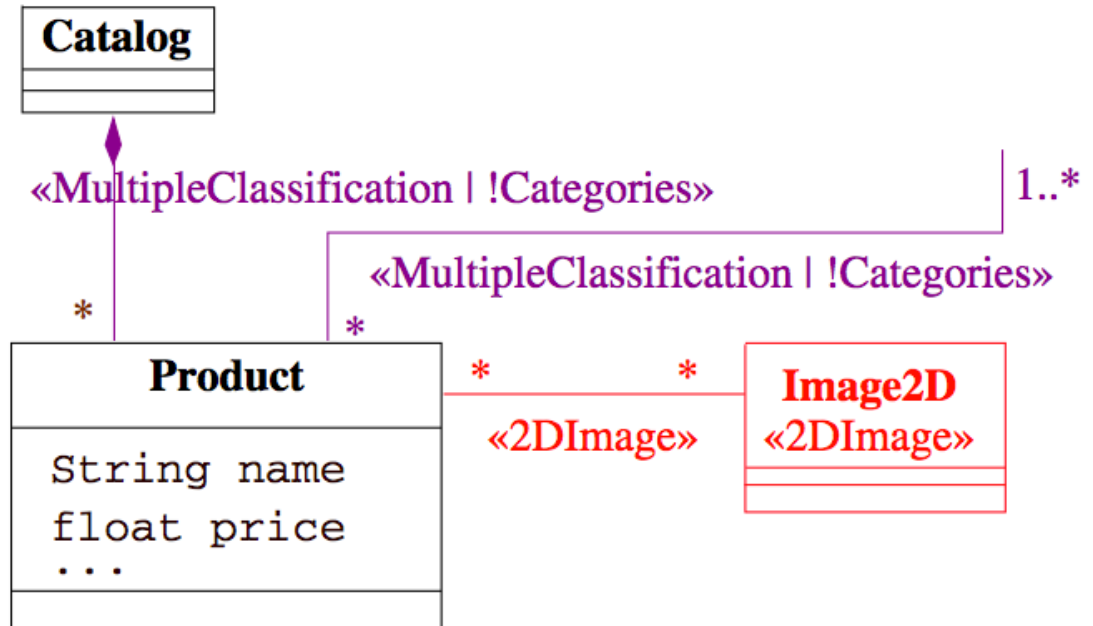
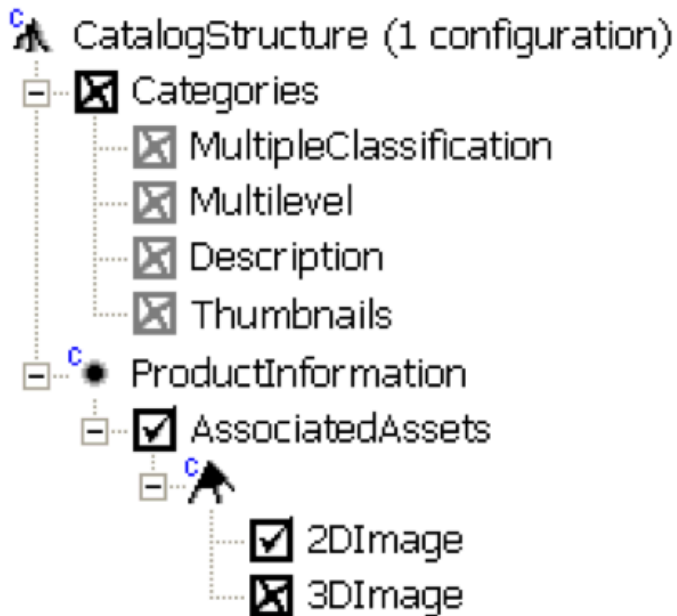
#ifdef WORLD
char * msg = "Hello_World\n";
#endif
#ifdef BYE
char * msg = "Bye_bye!\n";
#endif

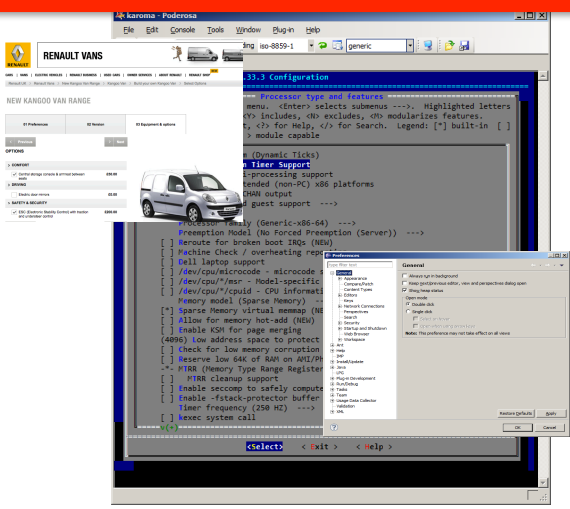
main() {
    printf(msg);
}
```





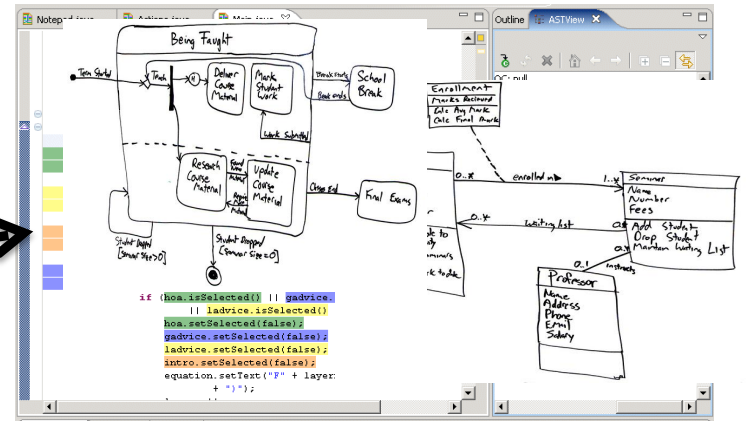
Ooops





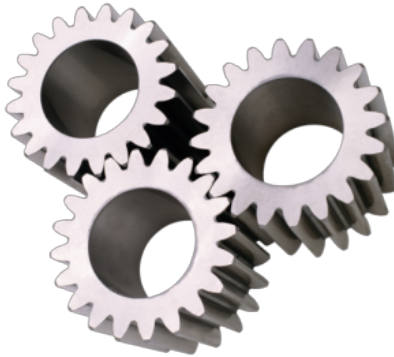
Variability Model

mapping



Base Artefacts (e.g., models)

Configuration



Software Generator (derivation engine)



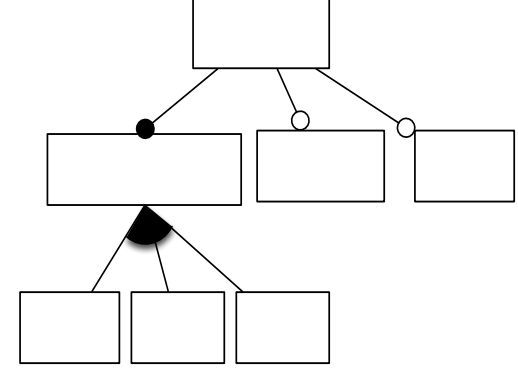
A vintage, rusted green truck is abandoned in a field of tall grass and brush. The truck is heavily weathered, with significant rust and missing parts, particularly the front end. The text "Unused flexibility" is overlaid in red on the truck's body.

Unused flexibility



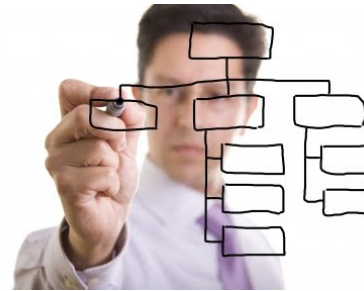
Illegal variant

Feature Model

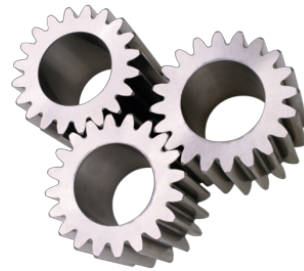


not, and, or, implies

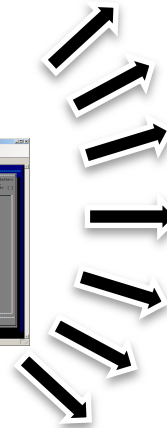
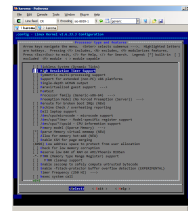
Communicative



Analytic

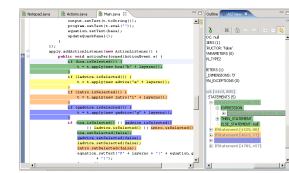
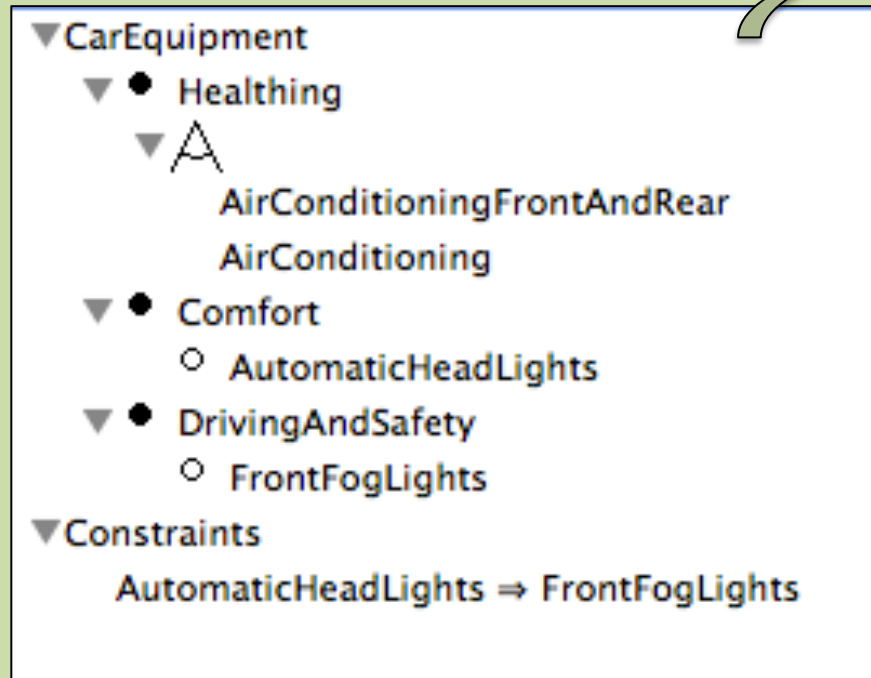


Generative



Feature Models

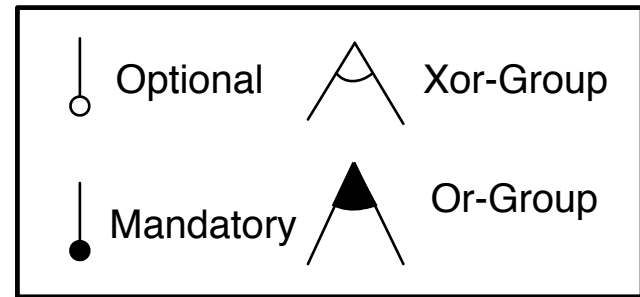
(defacto standard for modeling variability)

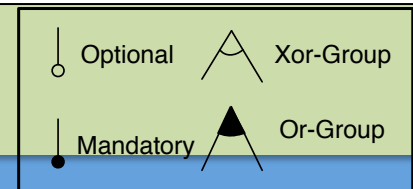
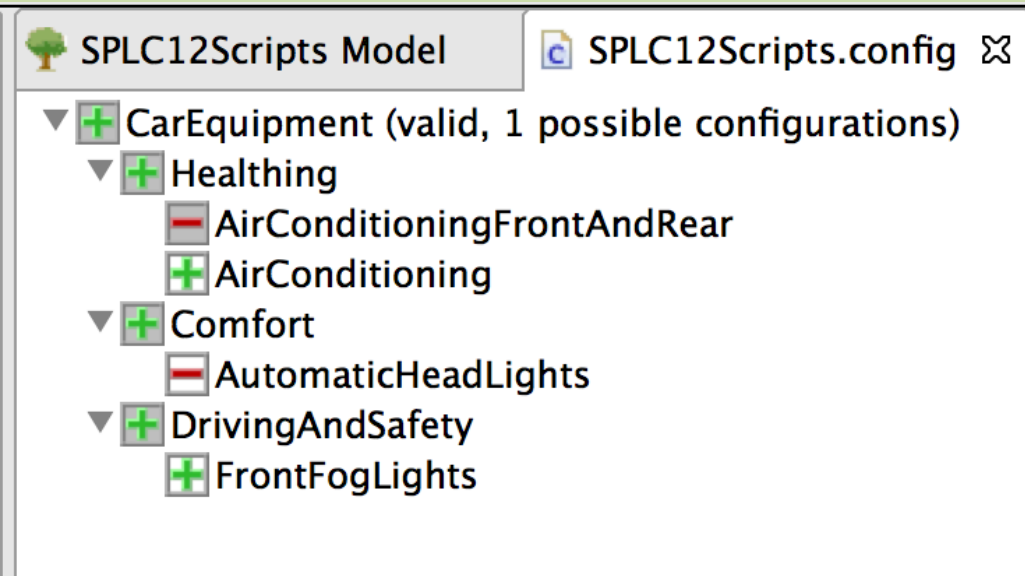


Hierarchy: rooted tree

Variability:

- mandatory,
- optional,
- Groups: exclusive or inclusive features
- Cross-tree constraints



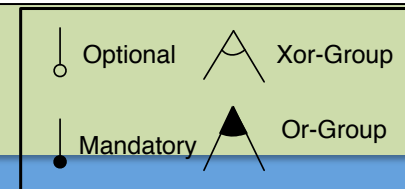
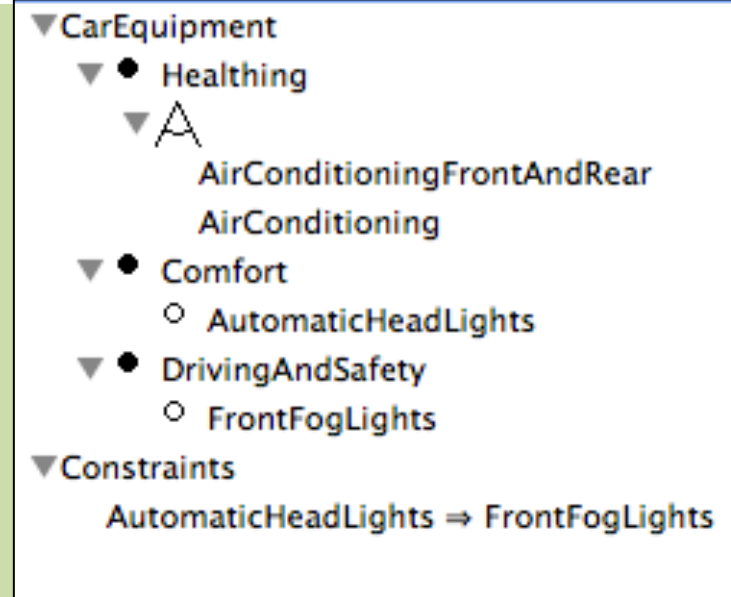
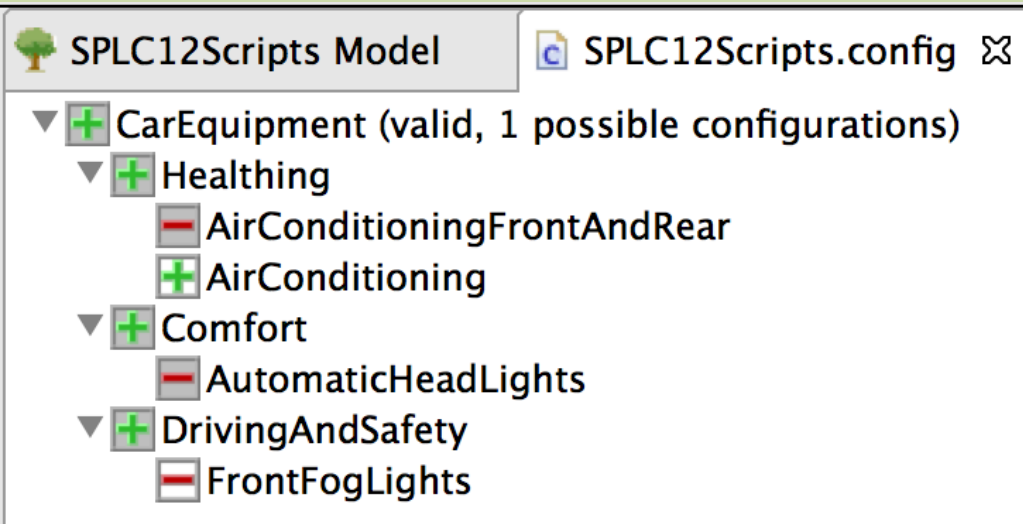


Hierarchy + Variability = set of valid configurations

configuration = set of features selected

{CarEquipment, Comfort, DrivingAndSafety, Healthing, AirConditioning, FrontFogLights}



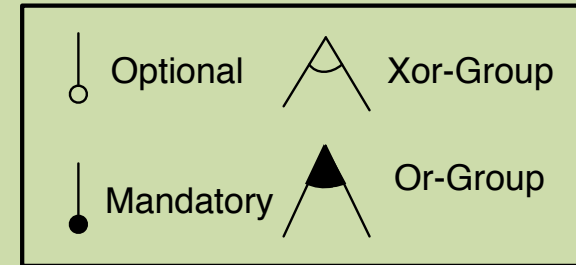
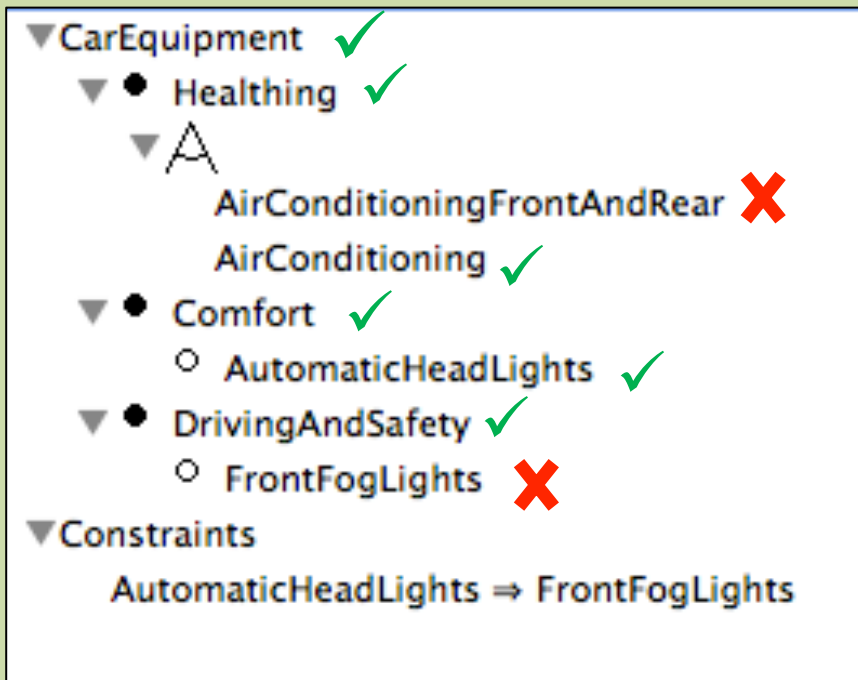


Hierarchy + Variability = set of valid configurations

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{CarEquipment, Comfort, DrivingAndSafety, Healthing, AirConditioning}



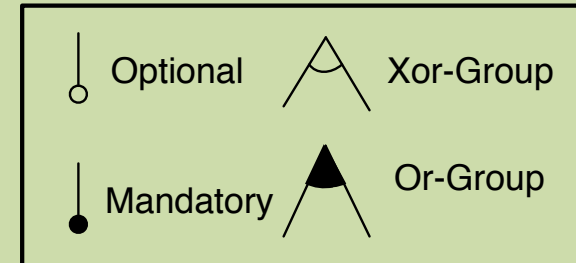
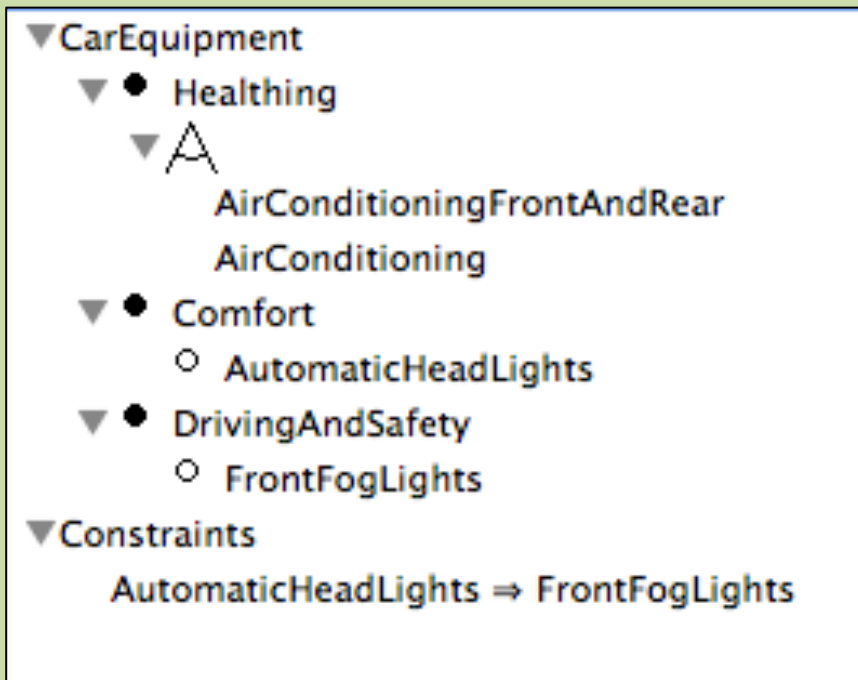


Hierarchy + Variability = set of valid configurations

configuration = set of features selected

{CarEquipment, Comfort, DrivingAndSafety, Healthing, AirConditioning, AutomaticHeadLights}





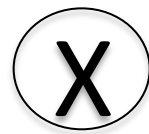
Hierarchy + Variability

=

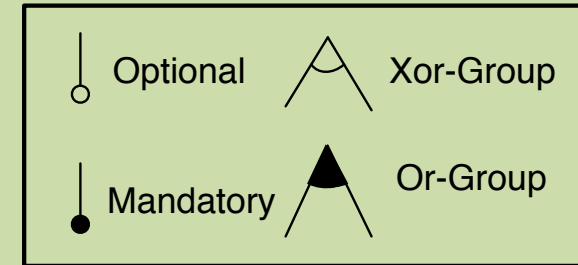
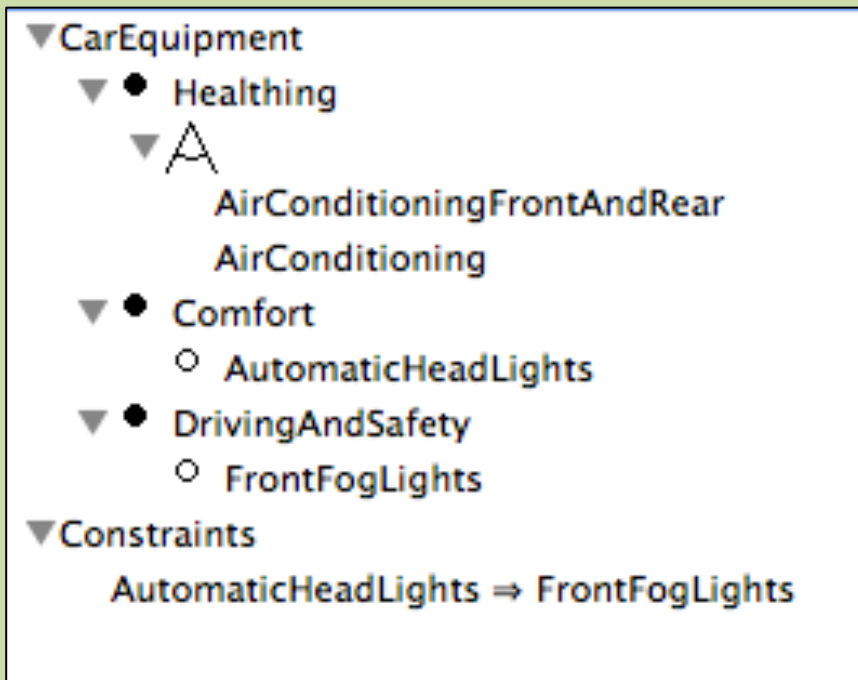
set of valid configurations



{CarEquipment, Comfort, DrivingAndSafety, Heating}



- {AirConditioning, FrontFogLights}
- {AutomaticHeadLights, AirConditioning, FrontFogLights}
- {AutomaticHeadLights, FrontFogLights, AirConditioningFrontAndRear}
- {AirConditioningFrontAndRear}
- {AirConditioning}
- {AirConditioningFrontAndRear, FrontFogLights}



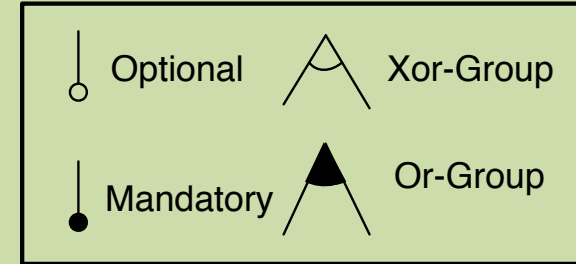
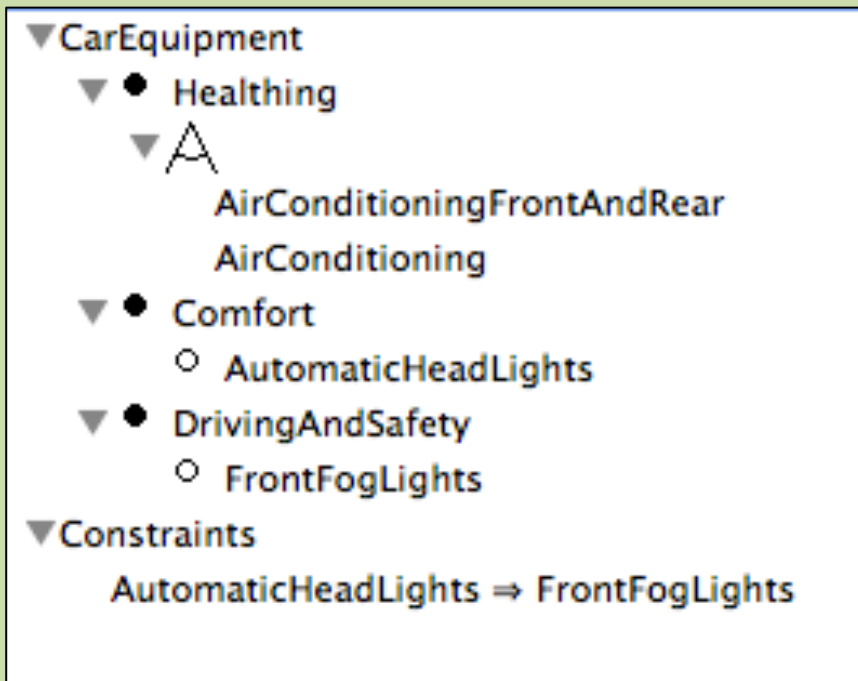
Hierarchy + Variability = set of valid configurations



Configuration set (from a basic feature model of car)

	CarEquipment	Comfort	DrivingAndSafety	Heating	AirConditioning	FrontFogLights	AutomaticHeadLights	AirConditioningFrontAndRear
{	Car2	yes	yes	yes	yes	yes	yes	no
[Car6	yes	yes	yes	yes	no	no	yes
	Car1	yes	yes	yes	yes	yes	no	no
	Car4	yes	yes	yes	yes	no	no	yes
	Car5	yes	yes	yes	yes	no	no	no
	Car3	yes	yes	yes	yes	no	yes	yes

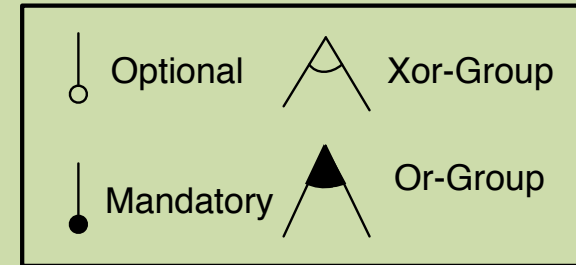
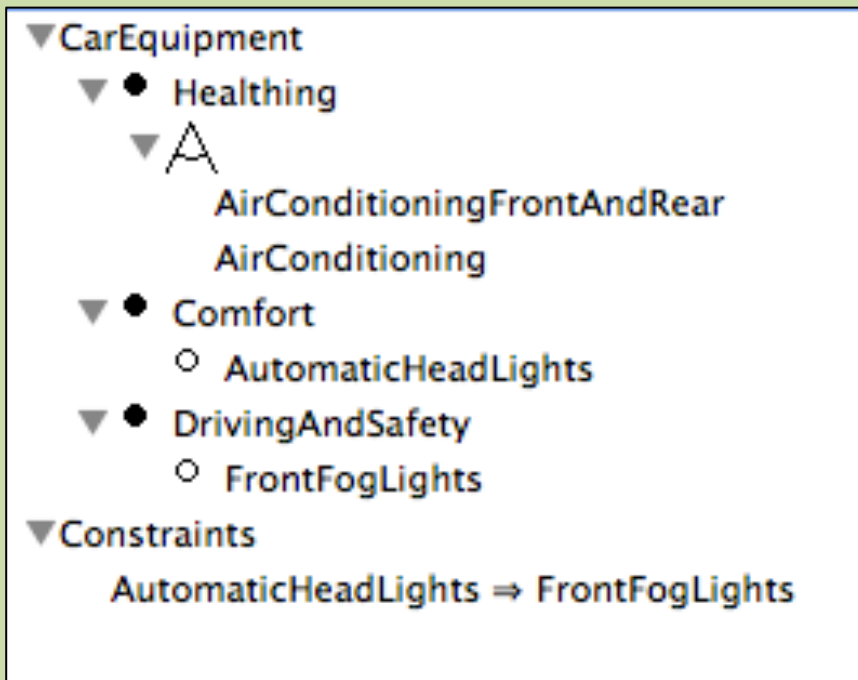
ar}



Hierarchy + Variability
 =
set of valid configurations



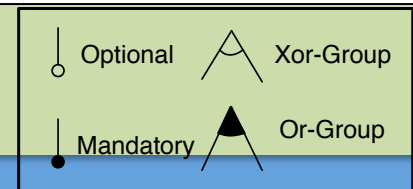
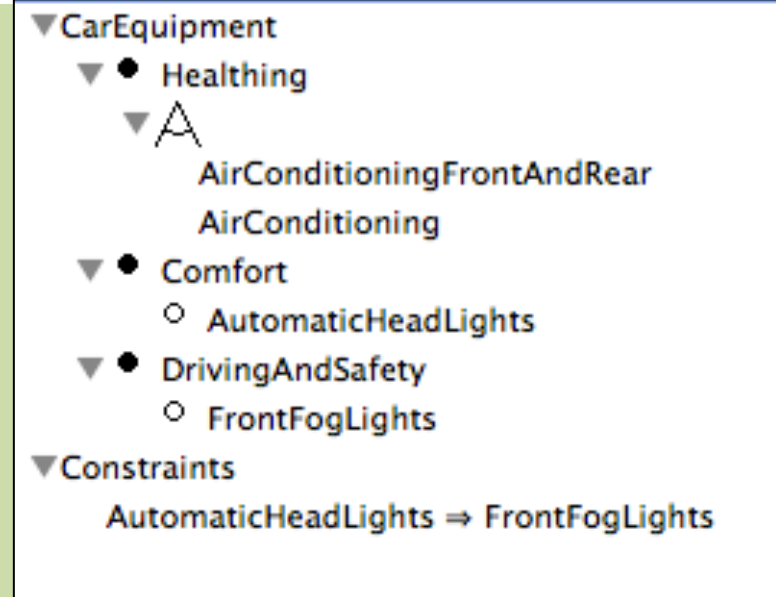
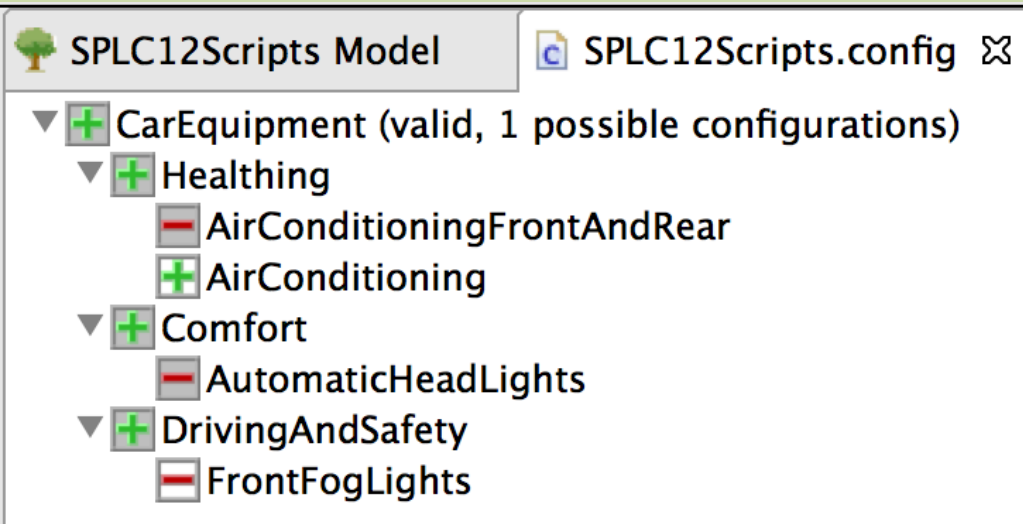
Product ▲ ▼	CarEquipment ▼	Comfort ▼	DrivingAndSafety ▼	Heating ▼	AirConditioning ▼	FrontFogLights ▼	AutomaticHeadLights ▼	AirConditioningFrontAndRear ▼
<input type="text" value="Find"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>
Car1	yes	yes	yes	yes	yes	yes	no	no
Car2	yes	yes	yes	yes	yes	yes	yes	no
Car3	yes	yes	yes	yes	no	yes	yes	yes
Car4	yes	yes	yes	yes	no	no	no	yes
Car5	yes	yes	yes	yes	yes	no	no	no
Car6	yes	yes	yes	yes	no	yes	no	yes



Hierarchy + Variability = set of valid configurations



Product ▲ ▼	▼	▼	▼	▼	AirConditioning ▼	FrontFogLights ▼	AutomaticHeadLights ▼	AirConditioningFrontAndRear ▼
Find <input type="text"/>					Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>
Car1					yes	yes	no	no
Car2					yes	yes	yes	no
Car3					no	yes	yes	yes
Car4					no	no	no	yes
Car5					yes	no	no	no
Car6					no	yes	no	yes

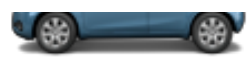


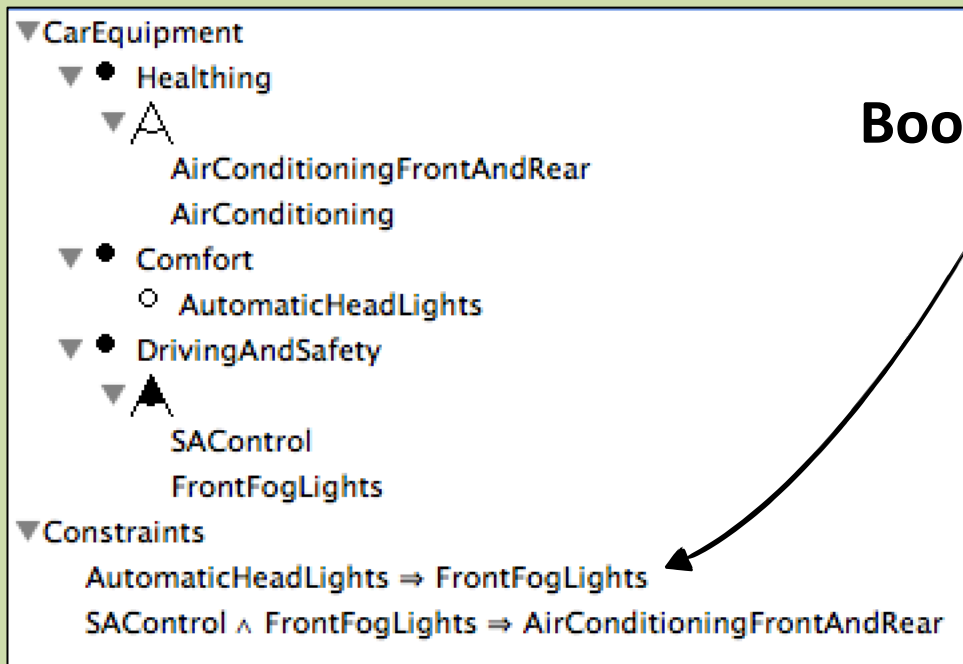
Hierarchy + Variability = set of valid configurations

configuration = set of features selected

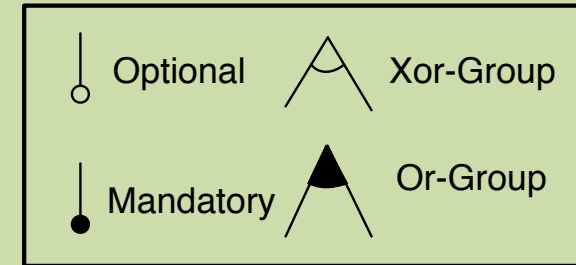
{CarEquipment, Comfort, DrivingAndSafety, Healthing, AirConditioning}

Product					AirConditioning	FrontFogLights	AutomaticHeadLights	AirConditioningFrontAndRear
Car5					Yes <input checked="" type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input checked="" type="checkbox"/>	Yes <input type="checkbox"/> No <input checked="" type="checkbox"/>	Yes <input type="checkbox"/> No <input checked="" type="checkbox"/>



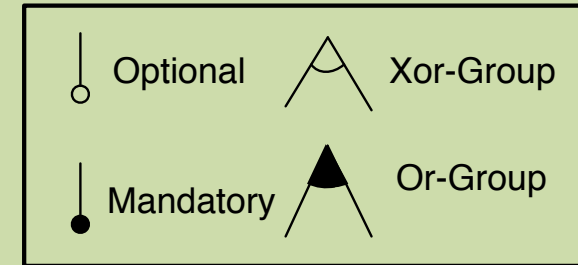
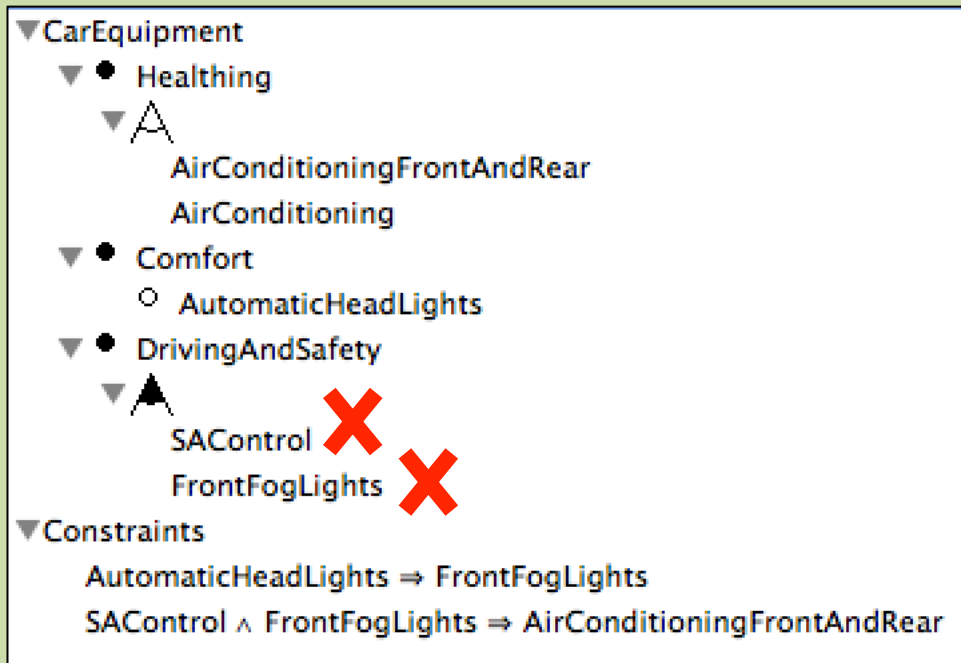


Boolean logic: \wedge , \vee , not, imple



Hierarchy + Variability
 =
set of valid configurations



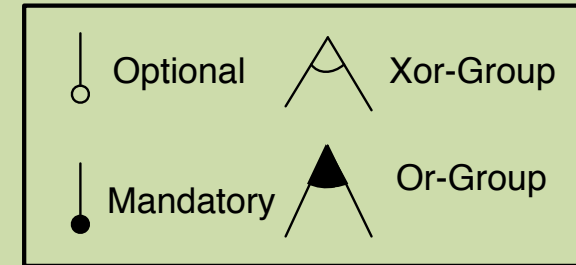
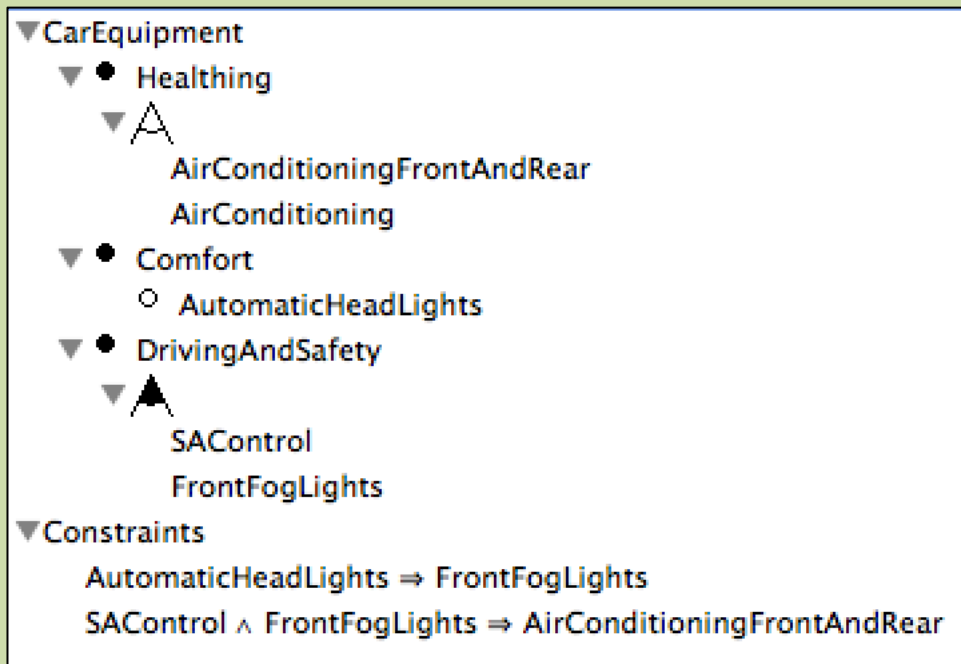


Hierarchy + Variability = set of valid configurations



Or-group: at least one!

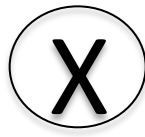




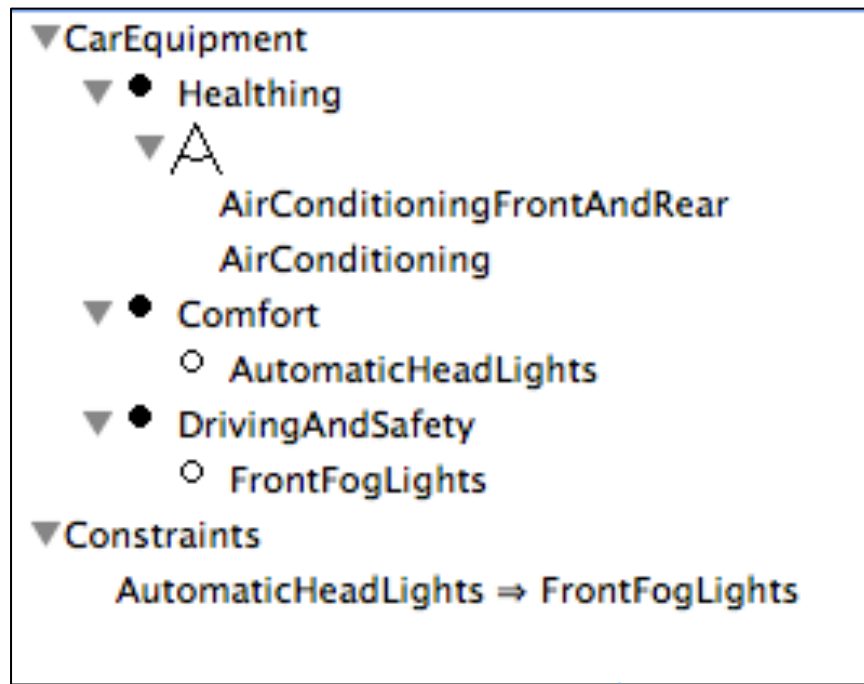
Hierarchy + Variability = set of valid configurations



{CarEquipment, Comfort, DrivingAndSafety, Healthing}



- {AirConditioningFrontAndRear, FrontFogLights, SAControl}
- {AirConditioningFrontAndRear, SAControl}
- {AutomaticHeadLights, AirConditioning, FrontFogLights}
- {AirConditioningFrontAndRear, SAControl, AutomaticHeadLights, FrontFogLights}
- {FrontFogLights, AirConditioning}
- {AutomaticHeadLights, AirConditioningFrontAndRear, FrontFogLights}
- {FrontFogLights, AirConditioningFrontAndRear}
- {SAControl, AirConditioning}



**(Boolean)
Feature Models**



**(Boolean)
Formula** Φ

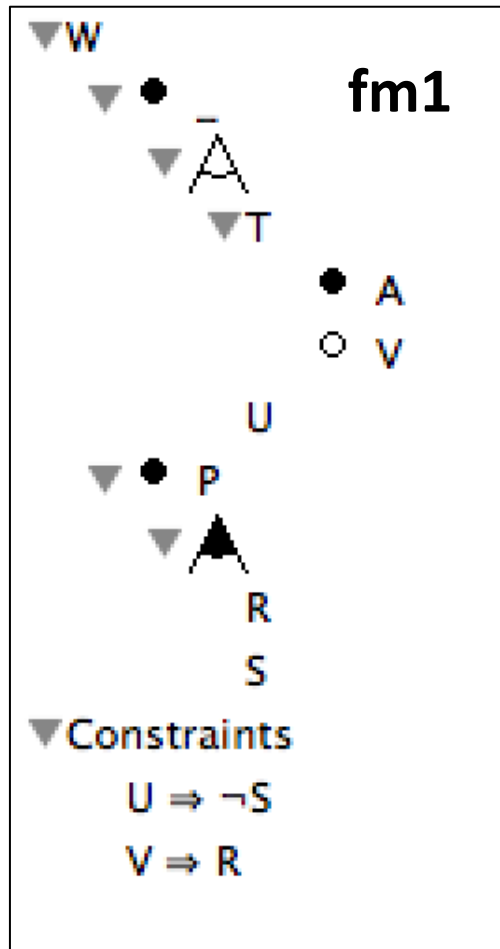


Product ▲	CarEquipment ▼	Comfort ▼	DrivingAndSafety ▼	Heating ▼	AirConditioning ▼	FrontFogLights ▼	AutomaticHeadLights ▼	AirConditioningFrontAndRear ▼
<input type="text" value="Find"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>	Yes <input type="checkbox"/> No <input type="checkbox"/>
Car1	yes	yes	yes	yes	yes	yes	no	no
Car2	yes	yes	yes	yes	yes	yes	yes	no
Car3	yes	yes	yes	yes	no	yes	yes	yes
Car4	yes	yes	yes	yes	no	no	no	yes
Car5	yes	yes	yes	yes	yes	no	no	no
Car6	yes	yes	yes	yes	no	yes	no	yes

**(Boolean)
Product Comparison Matrix**

(Boolean) Feature Models

Hierarchy + Variability = set of valid configurations



$$[[fm1]] = \{$$

$$\{W, P, R, S, T, A, V\},$$

$$\{W, P, S, T, A\},$$

$$\{W, P, R, T, A\},$$

$$\{W, P, R, U\},$$

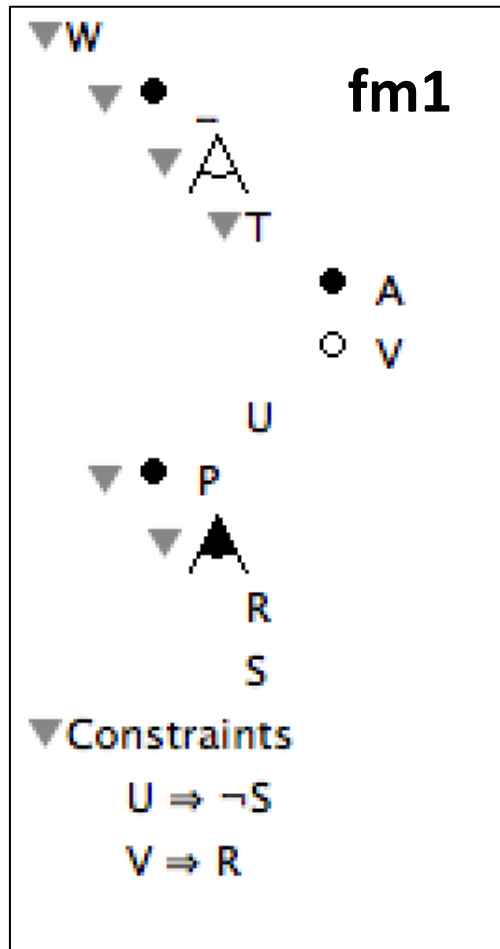
$$\{W, P, R, T, V, A\},$$

$$\{W, P, R, S, T, A\},$$

$$\}$$

(Boolean) Feature Models

~ Boolean formula

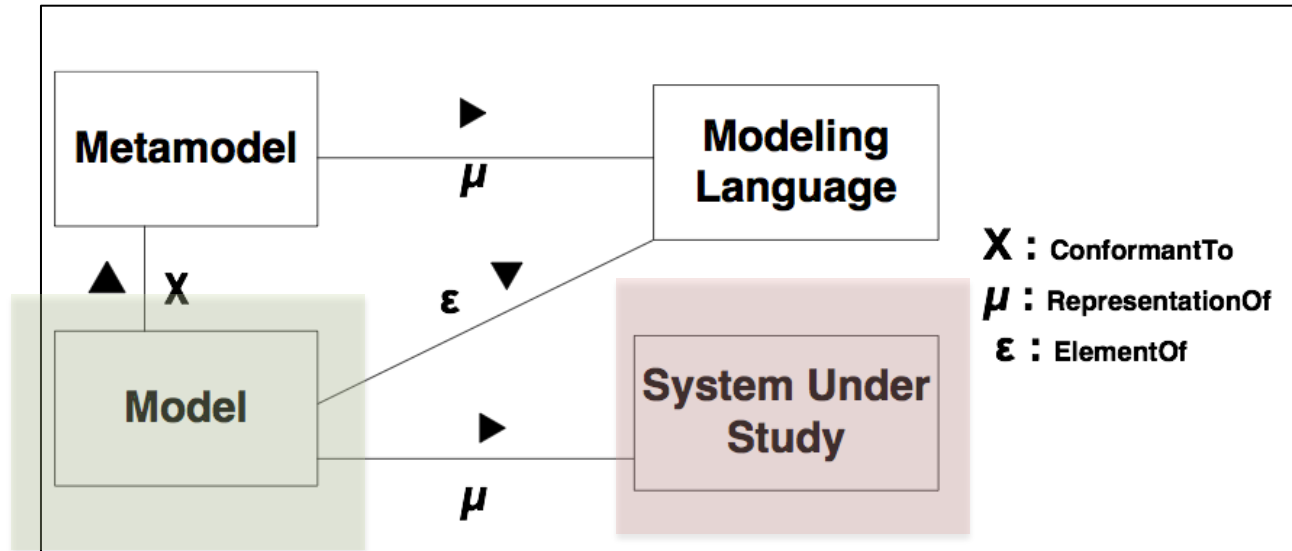


$\phi_{fm_1} = W // \text{root}$
 $\wedge W \Leftrightarrow P // \text{mandatory}$
 $// \text{Or-group}$
 $\wedge P \Rightarrow R \vee S$
 $\wedge R \Rightarrow P \wedge S \Rightarrow P$
 $\wedge V \Rightarrow T // \text{optional}$
 $\wedge A \Leftrightarrow T // \text{mandatory}$
 $// \text{Xor-group}$
 $\wedge T \Rightarrow W$
 $\wedge U \Rightarrow W$
 $\wedge \neg T \vee \neg U$
 $// \text{constraints}$
 $\wedge V \Rightarrow R // \text{implies}$
 $\wedge \neg U \Rightarrow \neg S // \text{excludes}$

Languages:

How to specify
feature models?

Feature Models



▼ CarEquipment

▼ ● Healinging



AirConditioningFrontAndRear

AirConditioning

▼ ● Comfort

○ AutomaticHeadLights

▼ ● DrivingAndSafety

○ FrontFogLights

▼ Constraints

AutomaticHeadLights ⇒ FrontFogLights

R8 Spyder
 5,2 FSI quattro R tronic
 Prix Total: 195.899,35 EUR
 Prix de base: 170.490,00 EUR
 Équipements optionnels: 25.409,35 EUR

Équipement	Prix
Châssis	320,65 EUR
Freins	931,70 EUR
Système d'aide au stationnement APS avant / arrière	1.373,35 EUR
Système d'aide au stationnement Advanced : APS avant et arrière et caméra arrière	1.790,80 EUR

Attention: Si une fonction est désactivée, elle n'est pas disponible.
 Un aperçu des Équipements:

Mode expert
 Mobile / Français

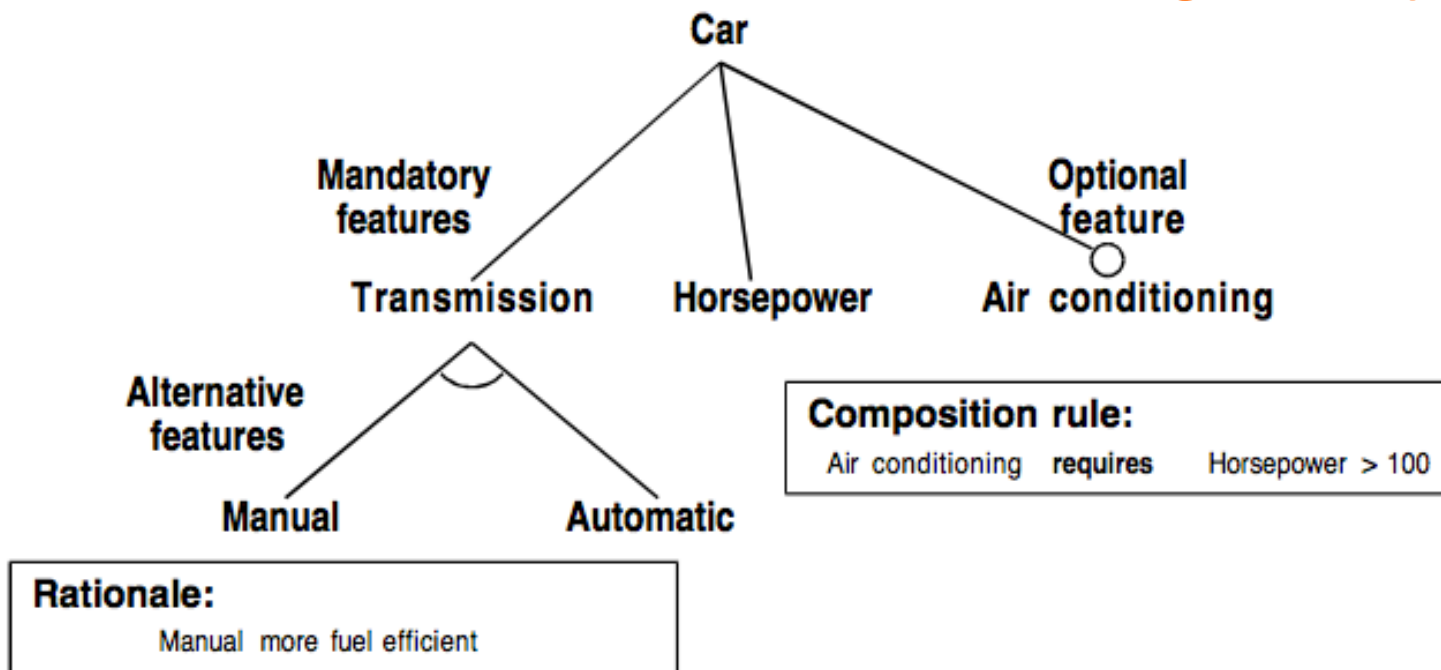
History of Feature Models

~ often called Feature Diagrams
(suggest a visual representation)

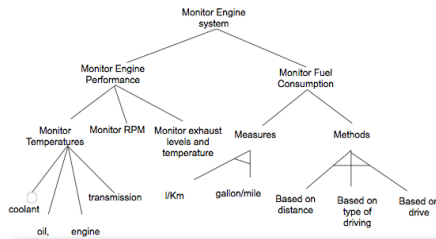
graphical languages

Feature Models

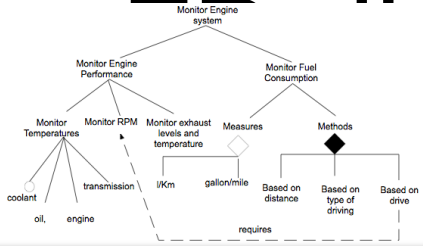
Kang et al. (1990)



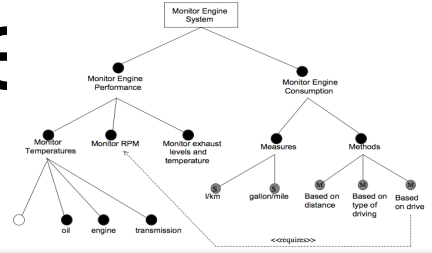
ale



FODA (OFT)
[Kang et al., 1990]



FeatuRSEB (RFD)
[Griss et al., 1998]



PLUSS (PFT)
[Eriksson et al., 2005]

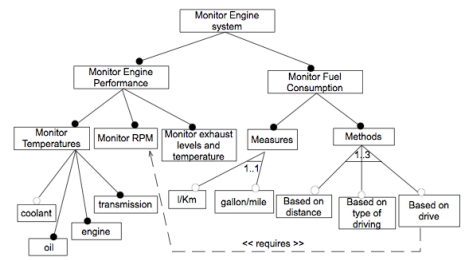
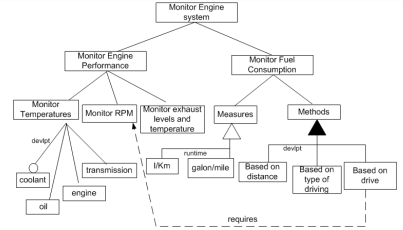
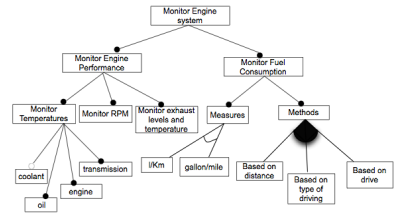
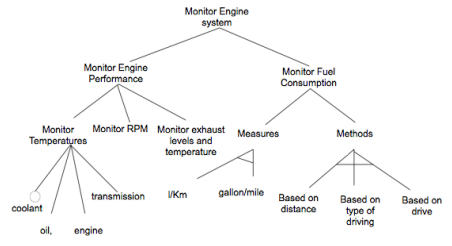


FORM (OFD)
[Kang et al., 1998]

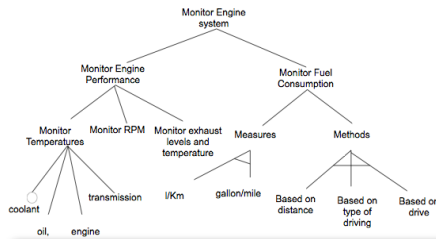
Gen. Prog. (GPFT)
[Czarnecki et al., 2000]

VBFD
[van Gorp et al., 2001]

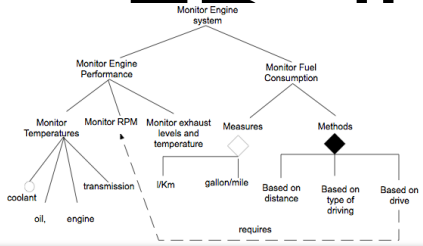
EFD
[Riebisch et al., 2002]



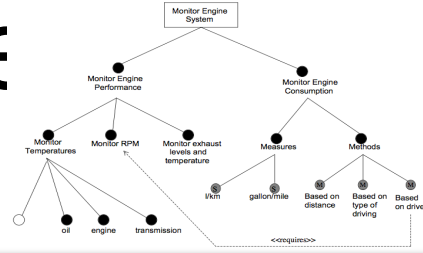
ale



FODA (OFT)
[Kang et al., 1990]



FeaturSEB (RFD)
[Griss et al., 1998]



PLUSS (PFT)
[Eriksson et al., 2005]

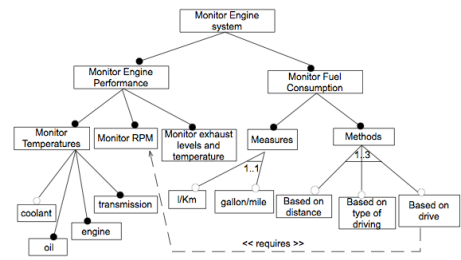
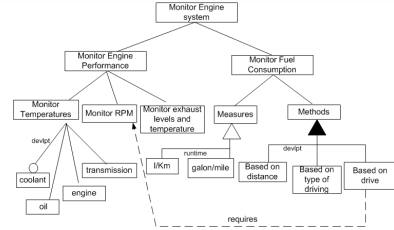
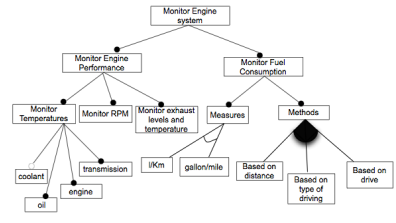
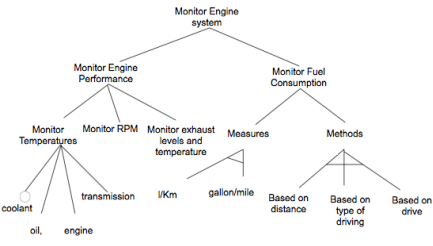
Aesthetic differences

FORM (OFD)
[Kang et al., 1998]

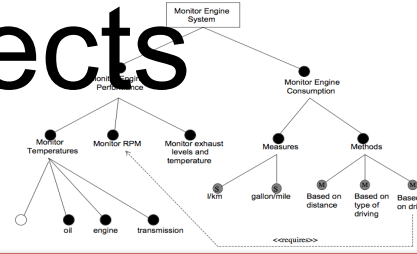
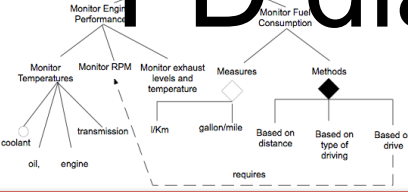
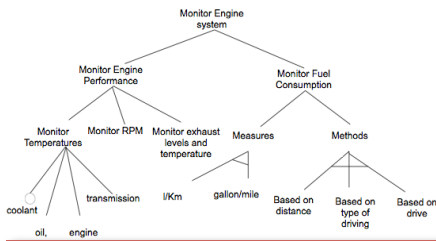
Gen. Prog. (GPFT)
[Czarnecki et al., 2000]

VBFD
[van Gorp et al., 2001]

EFD
[Riebisch et al., 2002]



FD dialects



FODA (OFT)
[Kang et al., 1990]

FeatURSEB (RFD)
[Griss et al., 1998]

PLUS (PFT)
[Eriksson et al., 2005]

Aesthetic differences

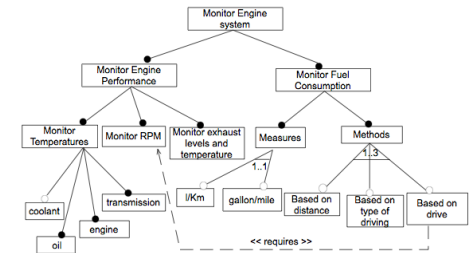
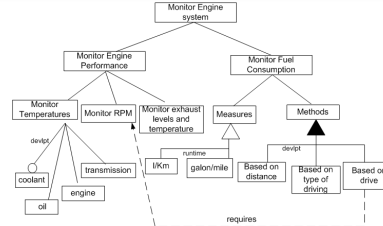
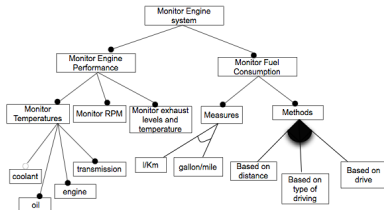
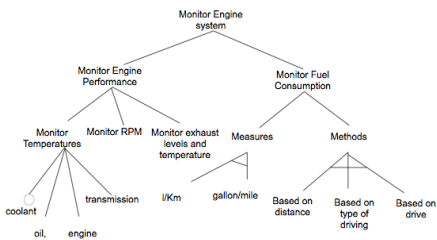
Stronger claims

FORM (OFD)
[Kang et al., 1998]

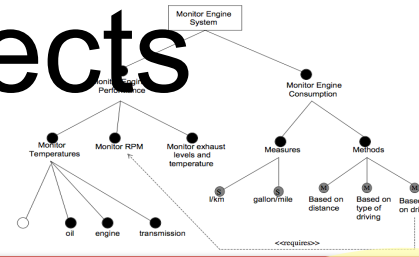
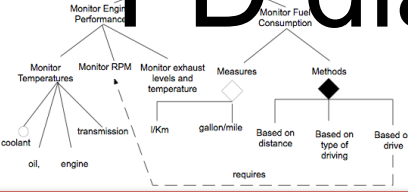
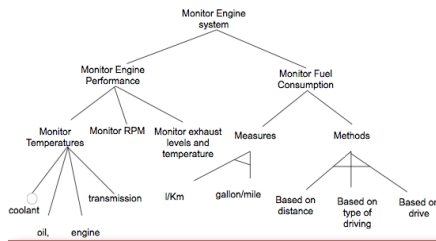
Gen. Prog. (GPFT)
[Czarnecki et al., 2000]

VBFD
[van Gorp et al., 2001]

EFD
[Riebisch et al., 2002]



FD dialects



FODA (OFT)
[Kang et al., 1990]

FeatURSEB (RFD)
[Griss et al., 1998]

PLUS (PFT)
[Eriksson et al., 2001]

Aesthetic differences

Stronger claims

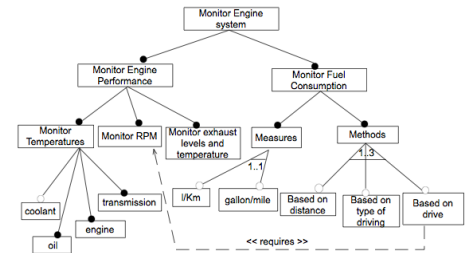
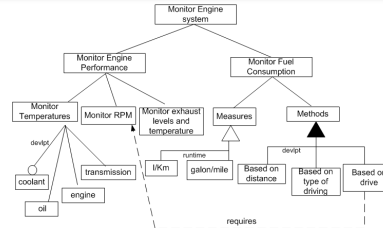
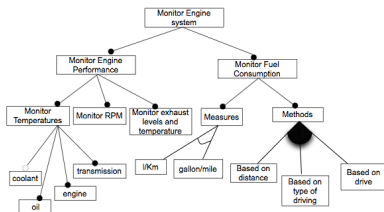
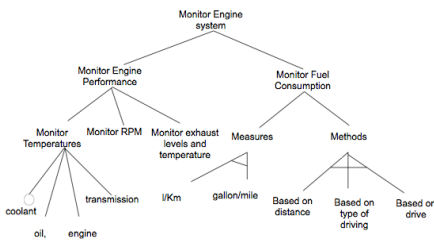
Vague use of terms
syntax, semantics, expressiveness, ambiguity, ...

FORM (OFD)
[Kang et al., 1998]

Gen. Prog. (GPFT)
[Czarnecki et al., 2000]

VBFD
[van Gorp et al., 2001]

EFD
[Riebisch et al., 2002]



Concrete (graphical)
syntax matters...

But semantics even more!

Feature Models

~ Feature Diagrams
(suggest a visual representation)

but that's not mandatory
that does not necessary scale
or respond to users' concerns

Textual variability languages

- Andreas Classen, Quentin Boucher, Patrick Heymans: A text-based approach to feature modelling: Syntax and semantics of TVL. *Sci. Comput. Program.* 76(12): 1130-1143 (2011)
- Czarnecki et al. “Clafer: Unifying Class and Feature Modeling” SoSyM’15
- Mathieu Acher, Philippe Collet, Philippe Lahire, Robert B. France « A Domain-Specific Language for Large-Scale Management of Feature Models » *Science of Computer Programming (SCP)*, 2013
- Mauricio Alférez, José A. Galindo, Mathieu Acher, and Benoit Baudry. *Modeling Variability in the Video Domain: Language and Experience Report* (2014). Research Report RR-8576

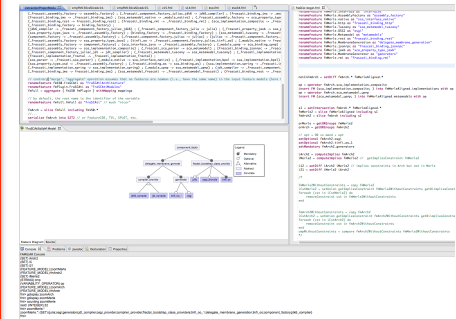
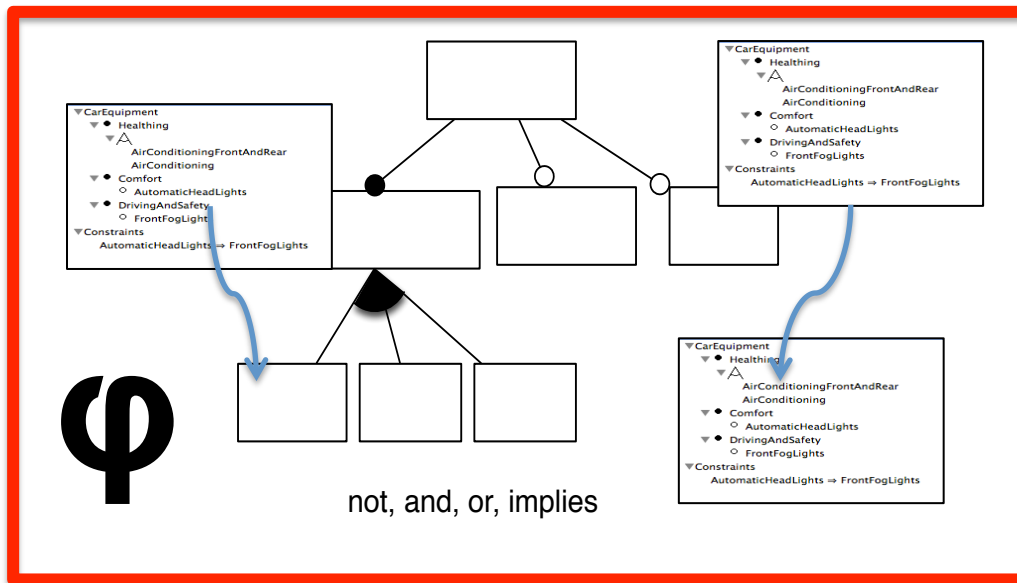
FAMiliAR

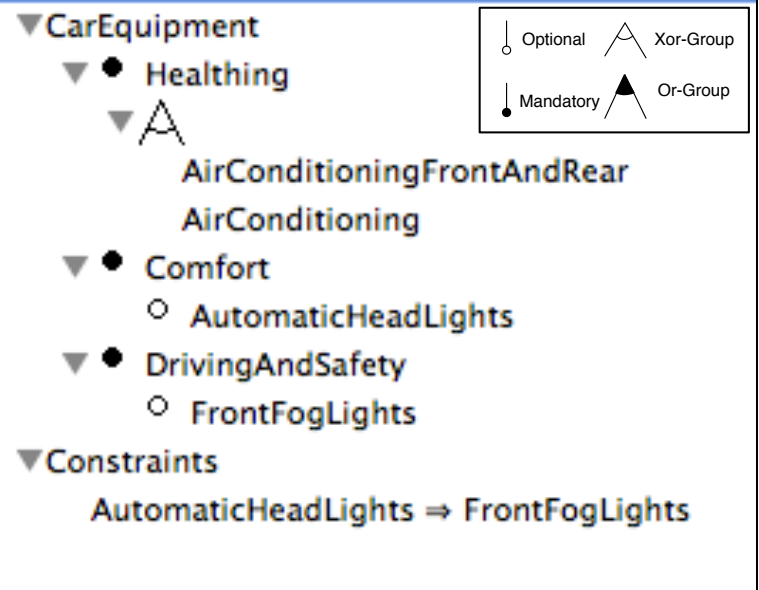
(FeAture Model script Language for manipulation and Automatic Reasoning)

<http://familiar-project.github.com/>



TVL
DIMACS





```
fml> convert fmCarEquipment into SPLOT
res1: (STRING) <feature_model name="fmCarEquipment">
<meta>
<data name="description"/>
<data name="creator"/>
<data name="address"/>
<data name="email"/>
<data name="phone"/>
<data name="website"/>
<data name="organization"/>
<data name="department"/>
<data name="date"/>
<data name="reference"/>
</meta>
<feature_tree>
:r CarEquipment(_r0)
  :m Healthing(_r1)
    :g [1,1]
      : AirConditioningFrontAndRear(_r2)
        : AirConditioning(_r3)
      :m DrivingAndSafety(_r4)
        :o FrontFogLights(_r5)
      :m Comfort(_r6)
        :o AutomaticHeadLights(_r7)
</feature_tree>
<constraints>
C0:~_r7 or _r5
</constraints>
</feature_model>
```



CarEquipment ;

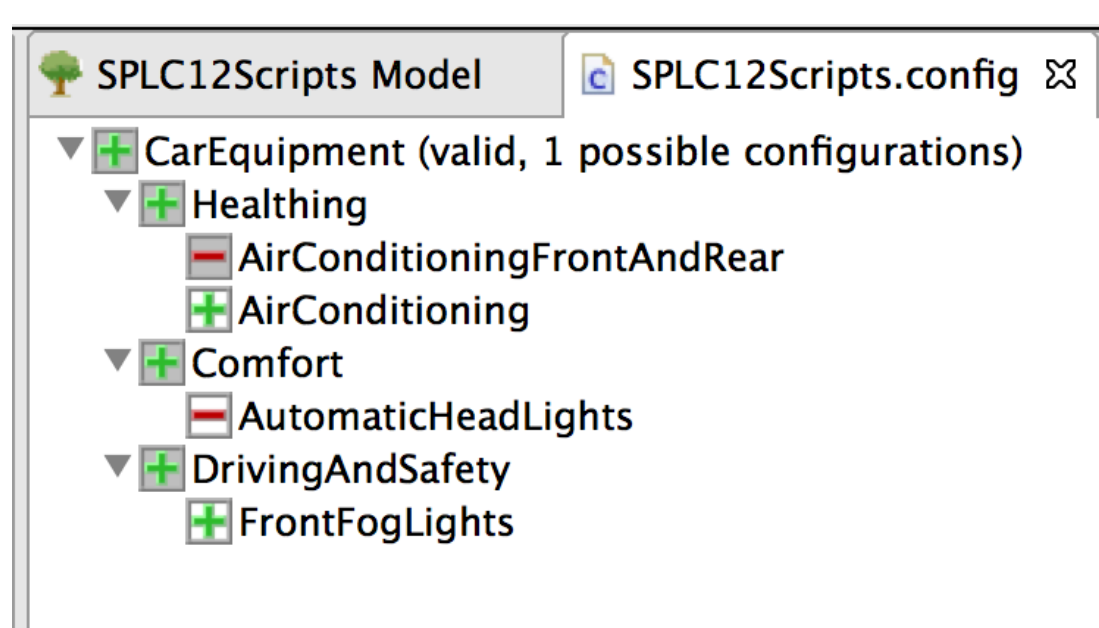
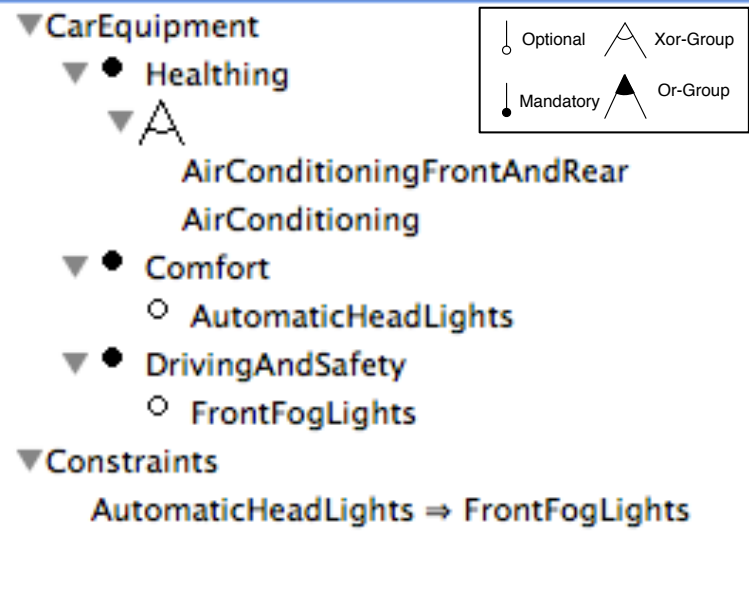
ty ;

IDE

eature

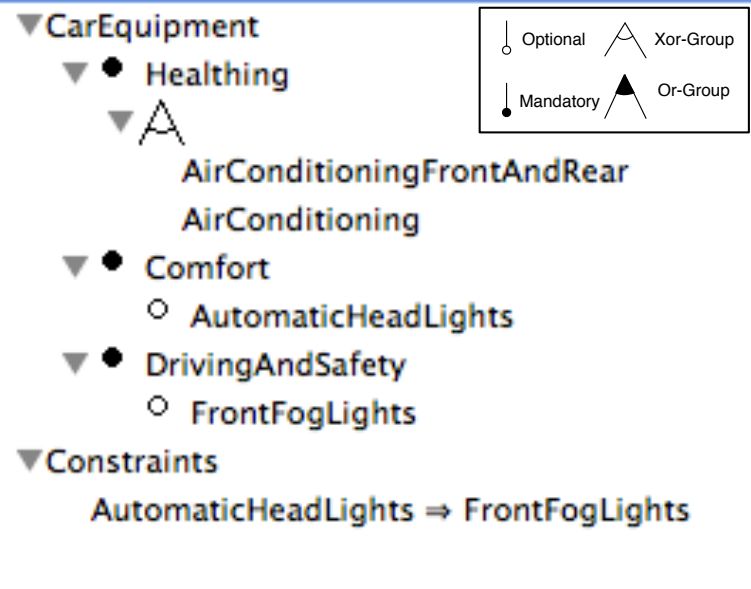
```
fmCarEquipment = FM (CarEquipment : Healthing DrivingAndSafety Comfort ; // 3 mandatory features
Healthing : (AirConditioning|AirConditioningFrontAndRear) ; // Xor
DrivingAndSafety : [FrontFogLights] ; // optional
Comfort : [AutomaticHeadLights] ; // optional
// cross-tree constraints
AutomaticHeadLights -> FrontFogLights ; )
```

```
MacBook-Pro-de-Mathieu-3:Documents macher1$ java -Xmx1024M -jar FML-basic-1.1.jar FMLTestRepository/carEquipTuto.fml
FAMILIAR (for FeAture Model scriPt Language for manIPulation and Automatic Reasoning) version 1.1 (beta)
http://familiar-project.github.com/
fml> ls
(FEATURE_MODEL) fmCarEquipment
fml> █
```

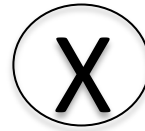


```
fmCarEquipment = FM (CarEquipment : Healthing DrivingAndSafety Comfort ; // 3 mandatory features
                    Healthing : (AirConditioning|AirConditioningFrontAndRear) ; // Xor
                    DrivingAndSafety : [FrontFogLights] ; // optional
                    Comfort : [AutomaticHeadLights] ; // optional
                    // cross-tree constraints
                    AutomaticHeadLights -> FrontFogLights ; )
```

```
fml> c1 = configuration fmCarEquipment
c1: (CONFIGURATION) selected: [Healthing, CarEquipment, DrivingAndSafety, Comfort]      deselected: []
fml> select AirConditioning FrontFogLights in c1
res2: (BOOLEAN) true
fml> deselect AutomaticHeadLights in c1
res3: (BOOLEAN) true
fml> selectedF c1
res4: (SET) {Comfort;CarEquipment;Healthing;AirConditioning;DrivingAndSafety;FrontFogLights}
```



{CarEquipment, Comfort,
DrivingAndSafety,
Healthing}



{AirConditioning, FrontFogLights}
 {AutomaticHeadLights, AirConditioning,
 FrontFogLights}
 {AutomaticHeadLights, FrontFogLights,
 AirConditioningFrontAndRear}
 {AirConditioningFrontAndRear}
 {AirConditioning}
 {AirConditioningFrontAndRear, FrontFogLights}

```
fmCarEquipment = FM (CarEquipment : Healthing DrivingAndSafety Comfort ; // 3 mandatory features
                    Healthing : (AirConditioning|AirConditioningFrontAndRear) ; // Xor
                    DrivingAndSafety : [FrontFogLights] ; // optional
                    Comfort : [AutomaticHeadLights] ; // optional
                    // cross-tree constraints
                    AutomaticHeadLights -> FrontFogLights ; )
```

```
fm> s1 = configs fmCarEquipment
s1: (SET) {{Comfort;CarEquipment;FrontFogLights;DrivingAndSafety;AirConditioning;Healthing};{AirConditioningFrontAndRear;CarEquipment;Comfort;DrivingAndSafety;Healthing};{FrontFogLights;DrivingAndSafety;AutomaticHeadLights;CarEquipment;Healthing;AirConditioningFrontAndRear;Comfort};{Healthing;CarEquipment;AirConditioning;DrivingAndSafety;Comfort};{Healthing;CarEquipment;Comfort;FrontFogLights;DrivingAndSafety;AirConditioningFrontAndRear};{AutomaticHeadLights;DrivingAndSafety;CarEquipment;AirConditioning;Healthing;Comfort;FrontFogLights}}
fm> size s1
res0: (INTEGER) 6
fm> counting fmCarEquipment
res1: (DOUBLE) 6.0
```

```
fm> co = cores fmCarEquipment
co: (SET) {CarEquipment;Healthing;DrivingAndSafety;Comfort}
fm> fmCarEquipment.*
res6: (SET) {DrivingAndSafety;AirConditioningFrontAndRear;Comfort;Healthing;FrontFogLights;AirConditioning;AutomaticHeadLights;CarEquipment}
fm> setDiff fmCarEquipment.* co
res7: (SET) {AutomaticHeadLights;FrontFogLights;AirConditioning;AirConditioningFrontAndRear}
fm>
```

```
fm1 = FM (A : B [C] ; B : E F ; C : (I|J) ; )
```

```
r1 = root fm1
```

```
s = children r1
```

```
s1 = children fm1.A
```

```
assert (s eq s1) // equality of the two sets
```

```
ft1 = parent fm1.F
```

```
str1 = name ft1
```

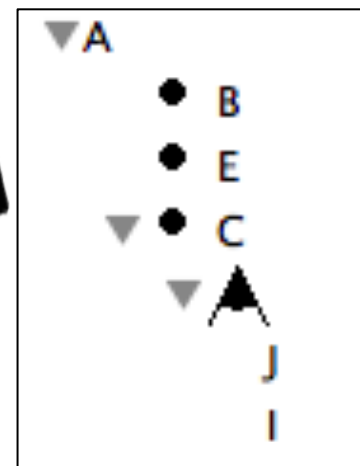
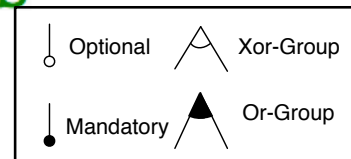
```
ft2 = parent F // parent fm1.F
```

```
// another FM
```

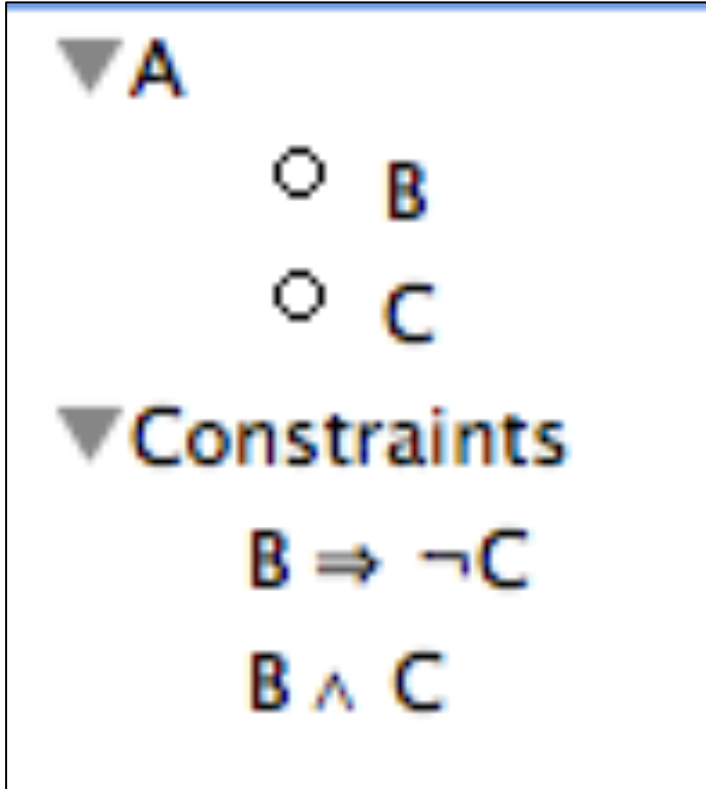
```
fm2 = FM (A : B C E ; C : (I|J)+ ; )
```

```
ft3 = fm2.B
```

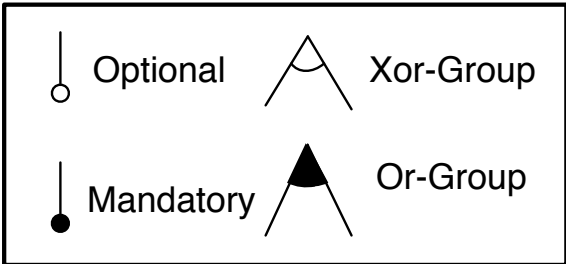
```
ft4 = name B // ambiguity
```

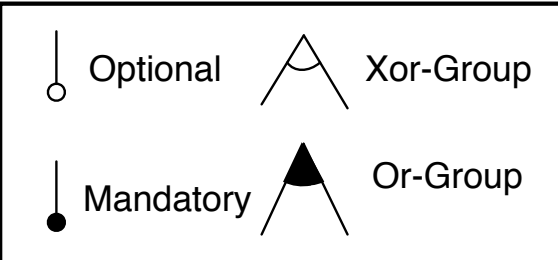
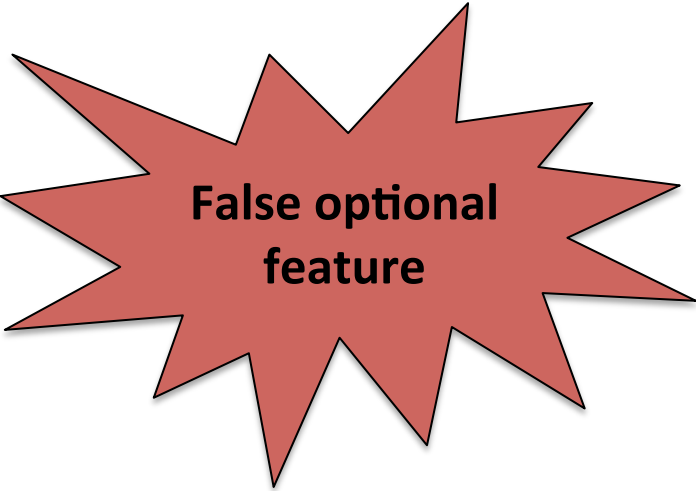
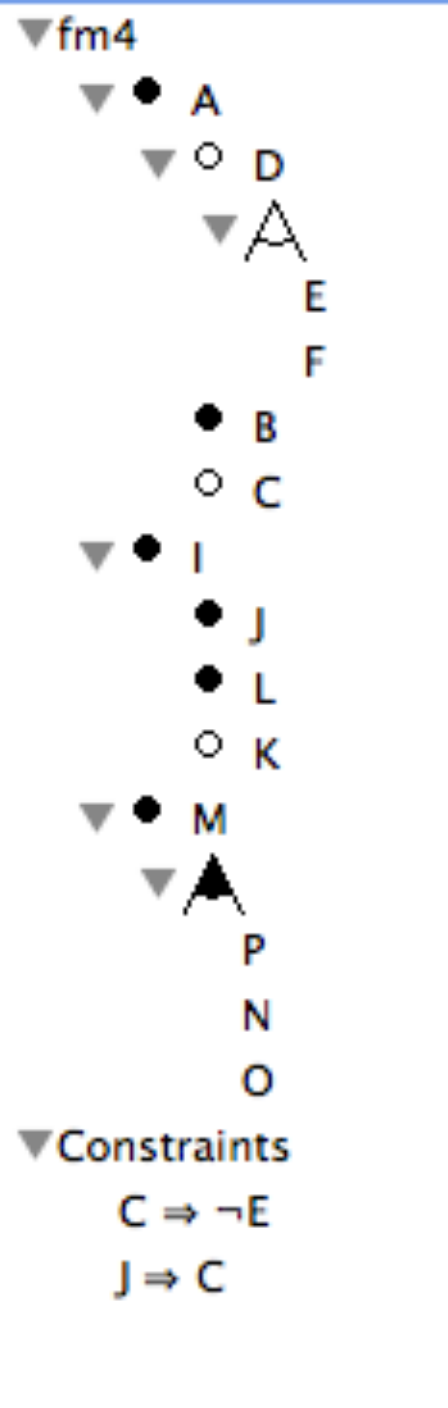


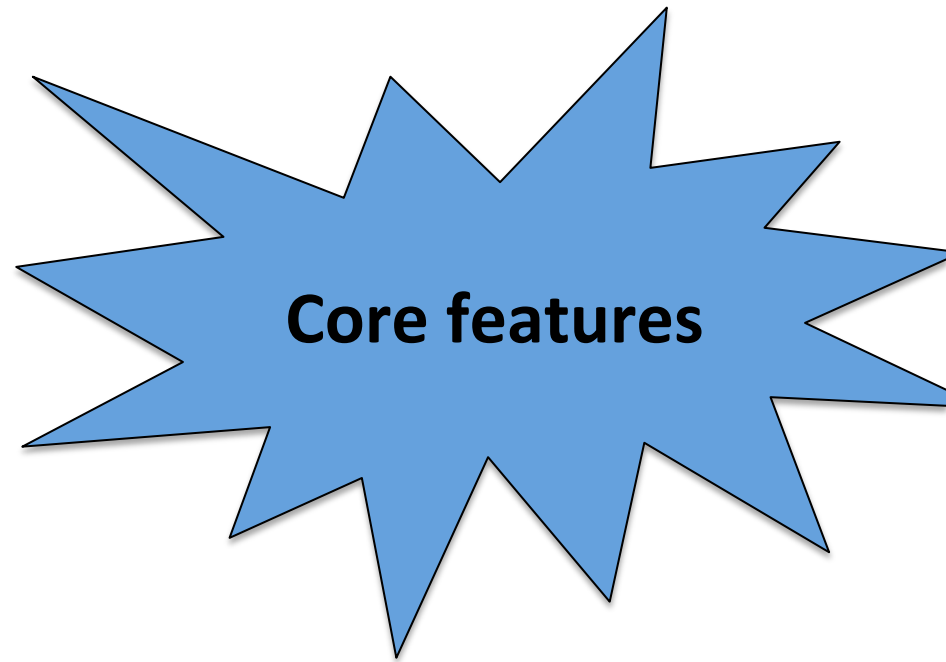
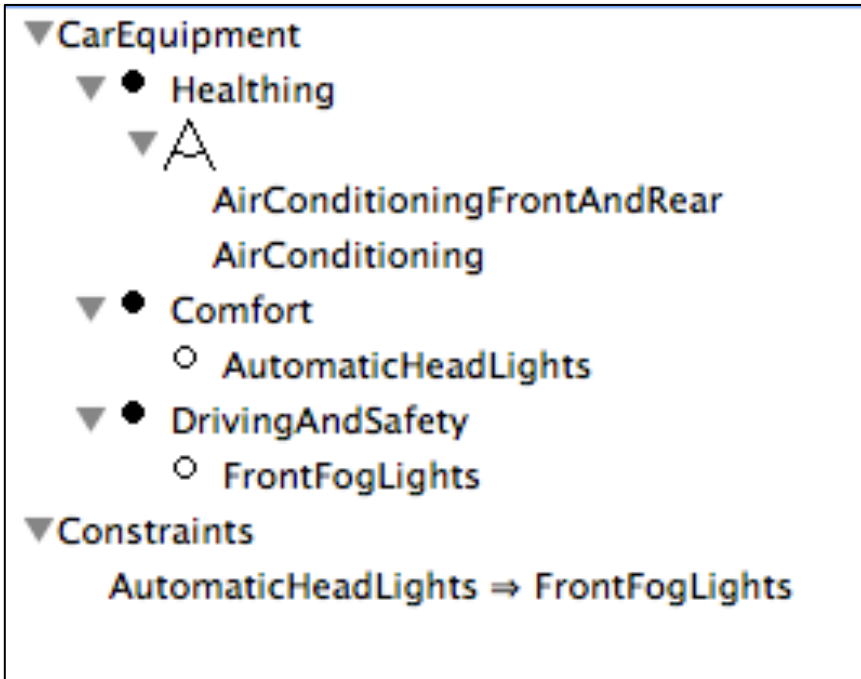
I want to analyze and
play with my specification!



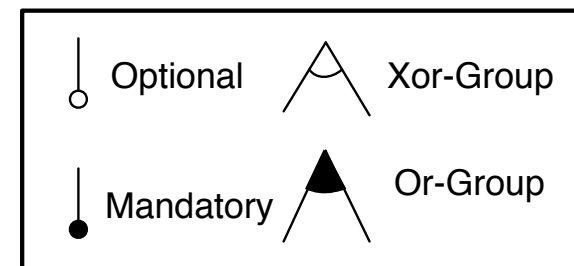
Empty set of configurations

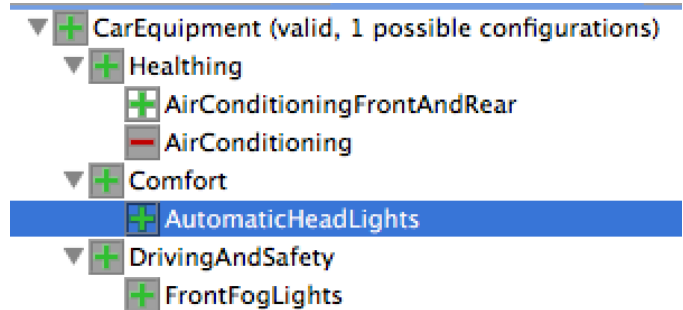
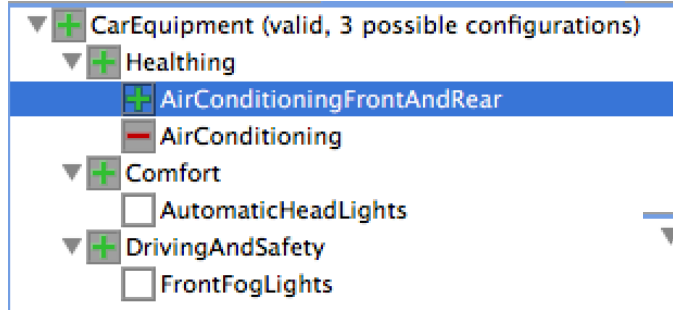
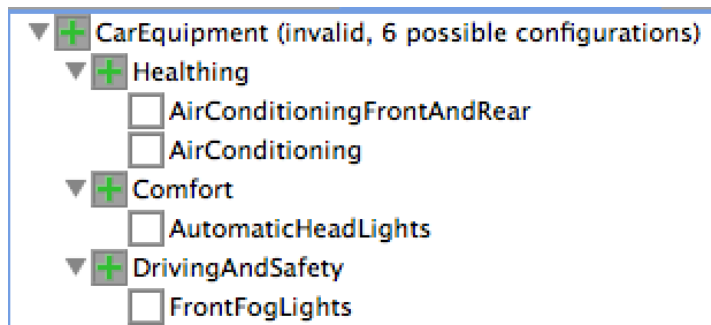
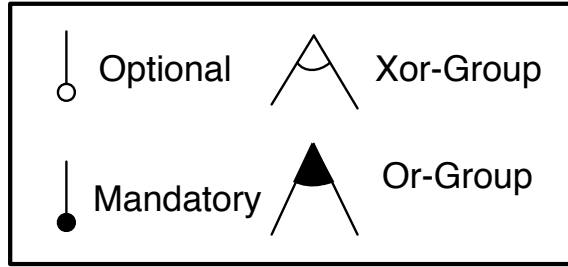
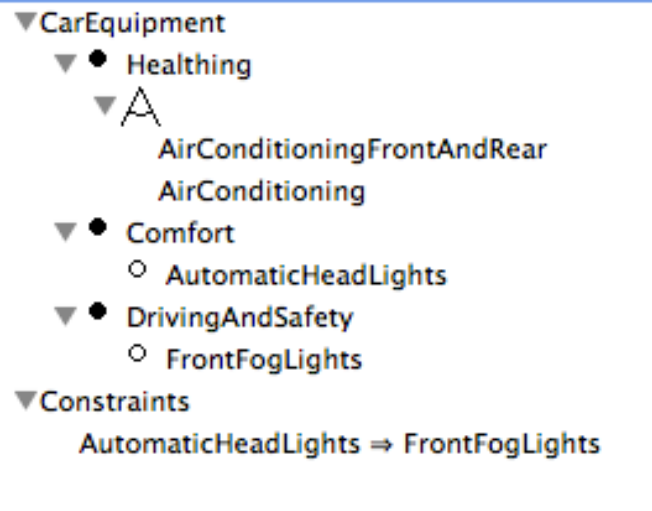






{CarEquipment, Comfort, DrivingAndSafety, Healthing}





Interactive Configuration

Feature Models and Automated Reasoning

Benavides et al. survey, 2010

	Batory [5]	Czamecki et al. [30]	Gheyi et al. [37]	Mannion et al. [51, 52]	Mendonca et al. [57]	Mendonca et al. [56]	Sun et al. [74]	Thüm et al. [75]	van der Storm [86, 87]	Zhang et al. [102, 101]	Zhang et al. [103]	Yan et al. [100]	Benavides et al. [10, 11, 12]	Benavides et al. [15]	Djebii et al. [34]	Trinidad et al. [78, 76]	White et al. [99]	White et al. [97]	Abo Zaid et al. [1]	Fan et al. [35]	Wang et al. [92, 93]	Benavides et al. [14]	Benavides et al. [16]	Segura [70]	Bachmeyer et al. [4]	Cao et al. [20]	Fernandez et al. [36]	Hemakumar [41]	Gheyi et al. [38]	Kang et al. [43]	Mendonca et al. [55]	Osman et al. [59, 60]	Salines et al. [66]	Van den Broek et al. [84]	Van Deursen et al. [88]	Von der Massen et al. [90]	Von der Massen et al. [91]	White et al. [98, 96]	Batory et al. [7]	Schobbens et al. [42, 68, 69]	Trinidad et al. [80]	Von der Massen et al. [89]						
	PL							CP					DL			Multi				Others										No support																		
Void feature model	+	+		+		+			+	+	+	+	+	+							+	+	+	+																								
#Products		+		⊕									+	+	+	+						+	+	+																								
Dead features		~				+				+	+	+		+					+							+																						
Valid product	+	+	+	+			⊕	+					⊕	⊕							+	+	+	+																								
All products	+	+	+	⊕			+	+					+														+																					
Explanations	+	~	+	⊕			⊕	+					⊕	⊕					+																													
Refactoring			+				⊕														+	+	+	+																								
Optimization							⊕						⊕	⊕							+	+	+	+																								
Commonality													⊕	⊕							+	+	+	+																								
Filter													⊕	⊕																																		
Valid partial configuration	+	+							+				⊕	⊕	+																																	
Atomic sets					+								⊕	⊕																																		
False optional features										⊕	⊕																																					
Corrective explanations											+																																					
Dependency analysis																																																
ECR					⊕	+																																										
Generalization			⊕		⊕			+					⊕																																			
Core features						+																																										
Variability factor													⊕																																			
Arbitrary edit								+																																								
Conditional dead features																																																
Homogeneity																																																
LCA					+																																											
Multi-step configuration																																																
Roots features					+																																											
Specialization								+																																								
Degree of orthogonality		~																																														
Redundancies																																																
Variant features																																																
Wrong cardinalities																																																
Feature model notation	B	C	B	B	B	B	B	B	B	B	C	B	B	C	C	B	B	B	B	B	B	B	C	B	B	B	B	B	B	B	B	C	C	C	C	B	B	B	B	B	B	C	C	C	B			
Extended feature model																																																
Formalization		+		+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	
	+	Supported		~	No support			⊕	Supported (first reference)				⊖	No support (first reference)			B	Basic feature model				C	Cardinality-based feature models																									

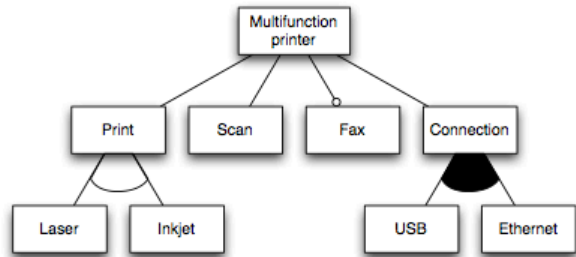
Decision problems and complexity

- Validity of a feature model
- Validity of a configuration
- Computation of dead and core features
- Counting of the number of valid configurations
- Equivalence between two feature models
- Satisfiability (SAT) problem
 - NP-complete

How to automate analysis of your feature models?

Binary Decision Diagram (BDD)
SAT solver

Typical implementations



result



logics



solvers



Z3

Truth table, boolean function

<i>from</i>		<i>to</i>		
x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

BDDs from Truth Tables

Truth Table



Binary Decision Tree



Binary Decision Diagram (BDD)



Ordered Binary Decision Diagram (OBDD)

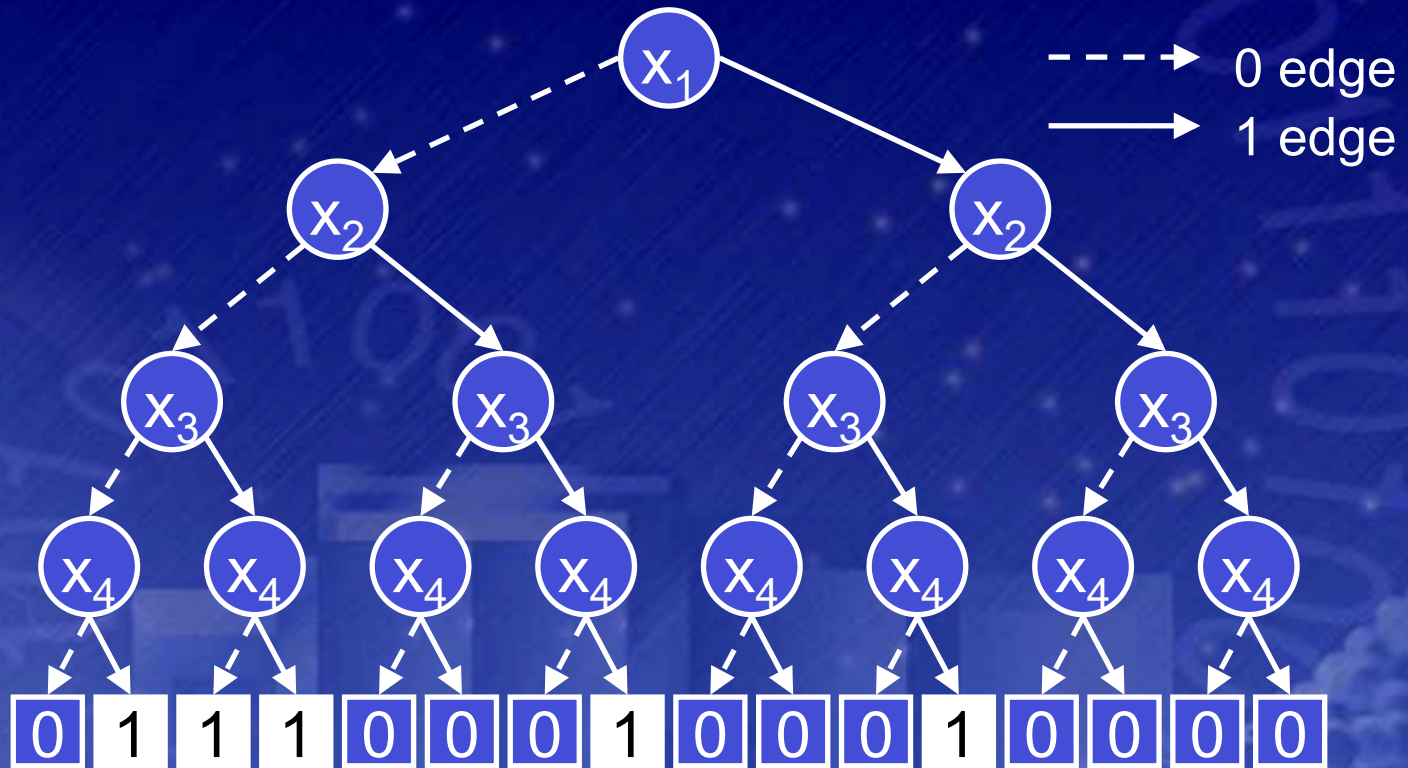


Reduced Ordered Binary Decision Diagram
(ROBDD, simply called BDD)

Binary Decision Diagrams (Bryant 1986)

encoding of a truth table.

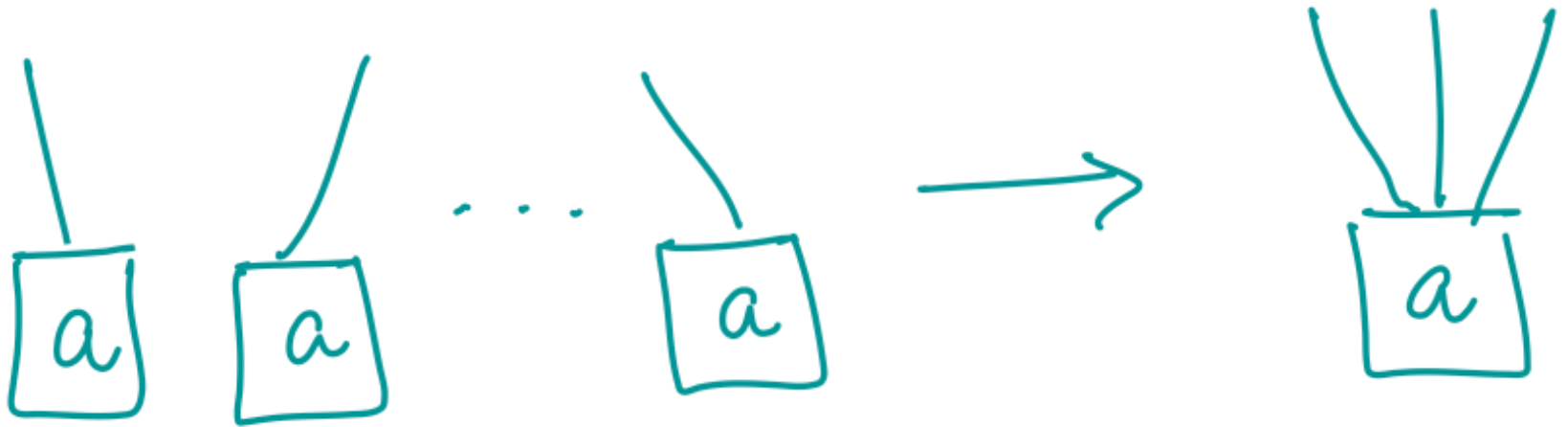
from		to		
x_1	x_2	x_3	x_4	f
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



Reduction

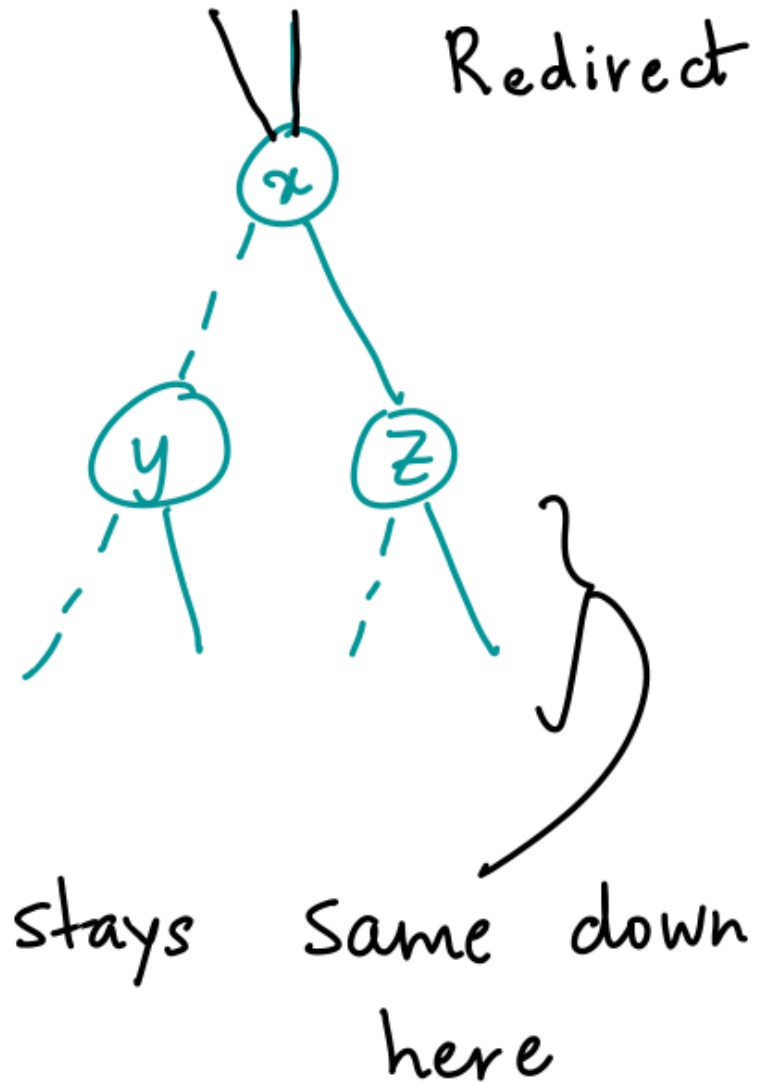
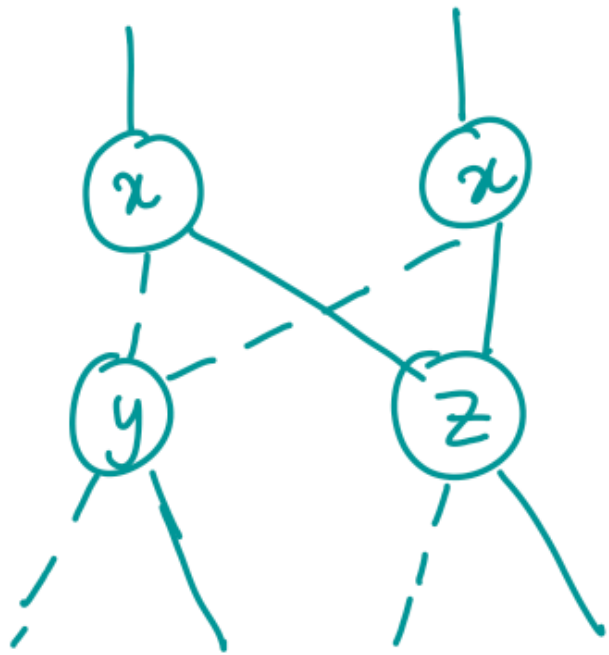
- Identify Redundancies
- 3 Rules
 - Merge equivalent leaves
 - Merge isomorphic nodes
 - Eliminate redundant tests

Merge equivalent leaves

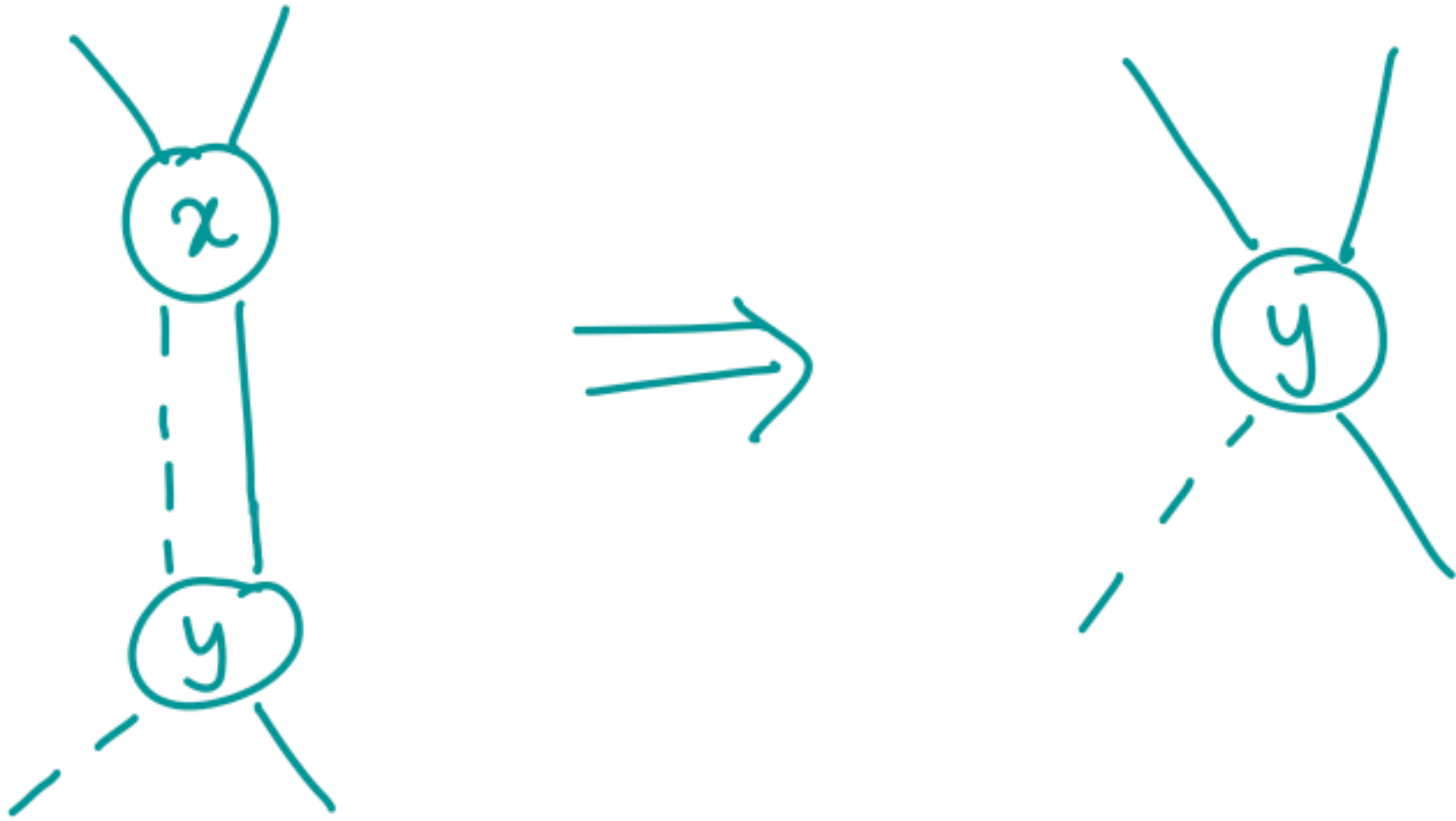


"a" is either 0 or 1

Merge isomorphic nodes

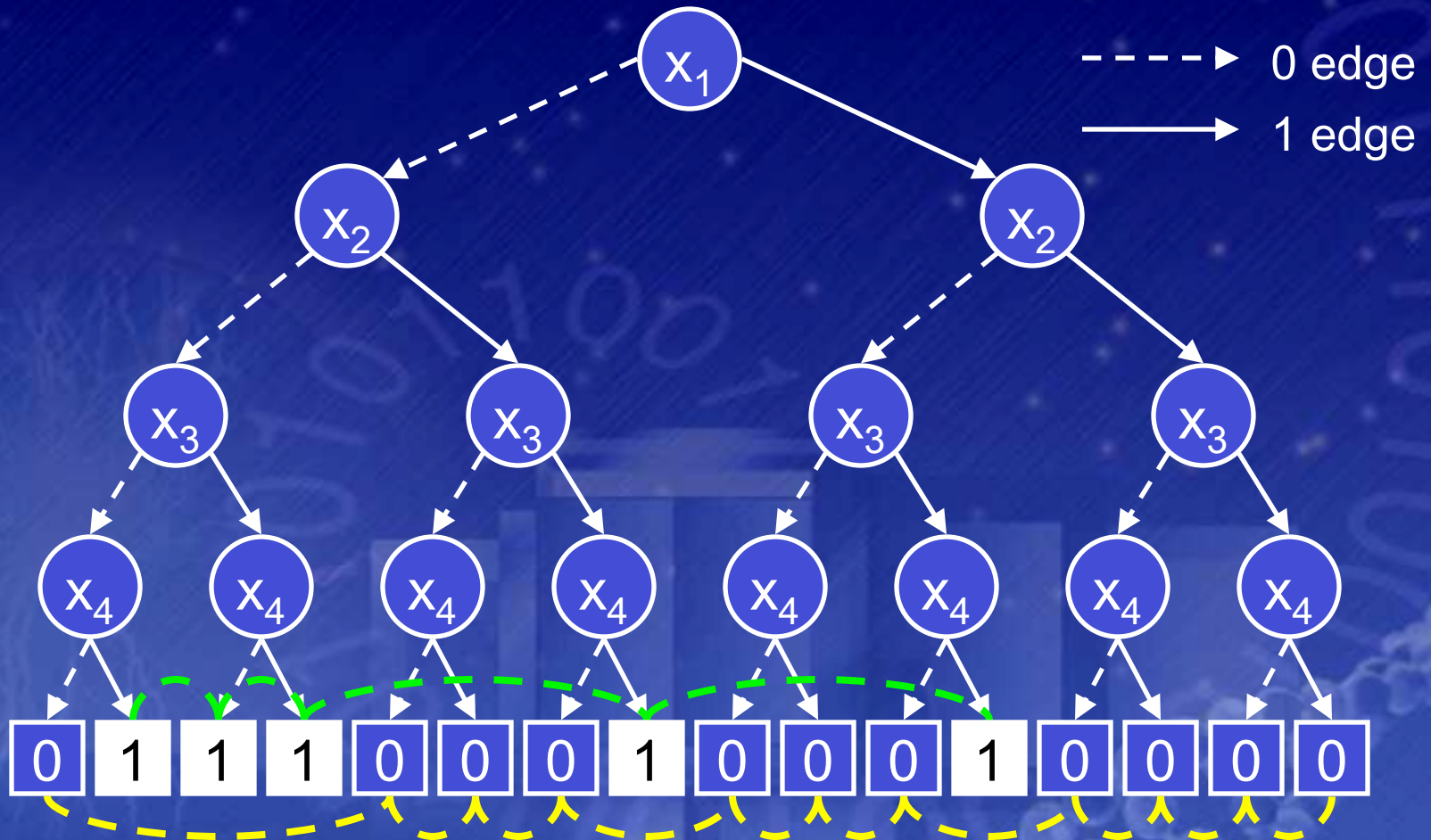


Eliminate redundant tests



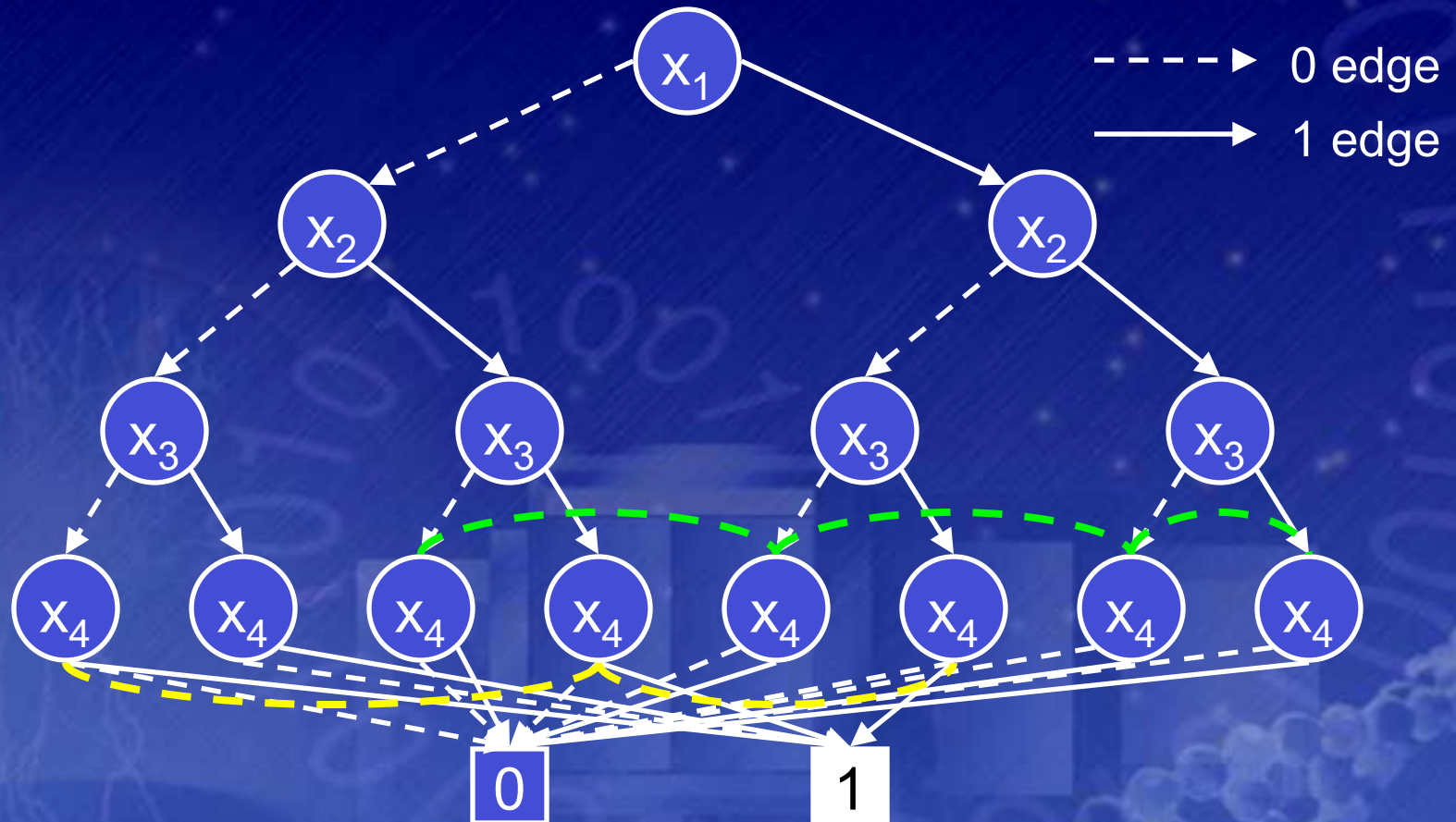
Binary Decision Diagrams

- Collapse redundant nodes.



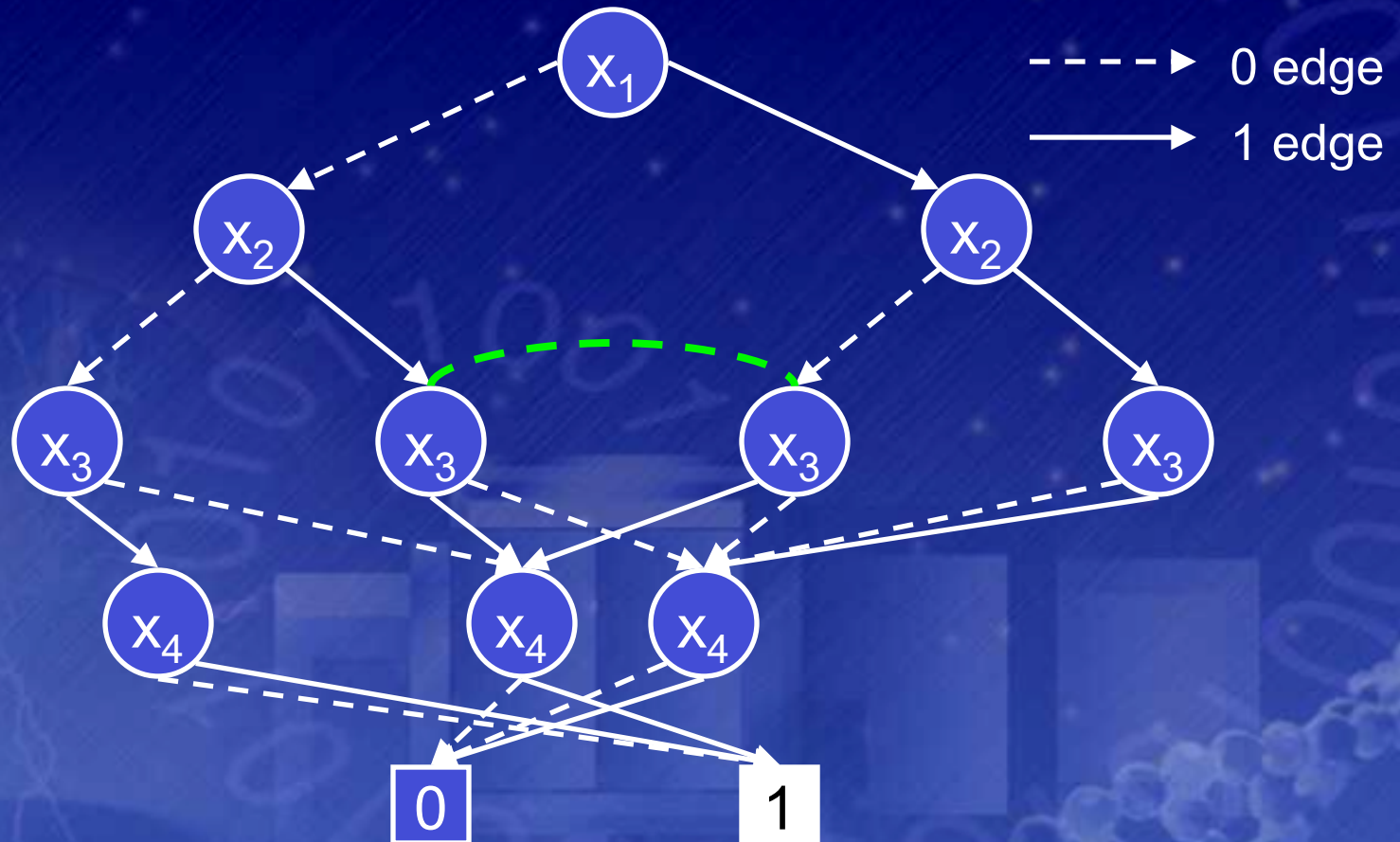
Binary Decision Diagrams

- Collapse redundant nodes.



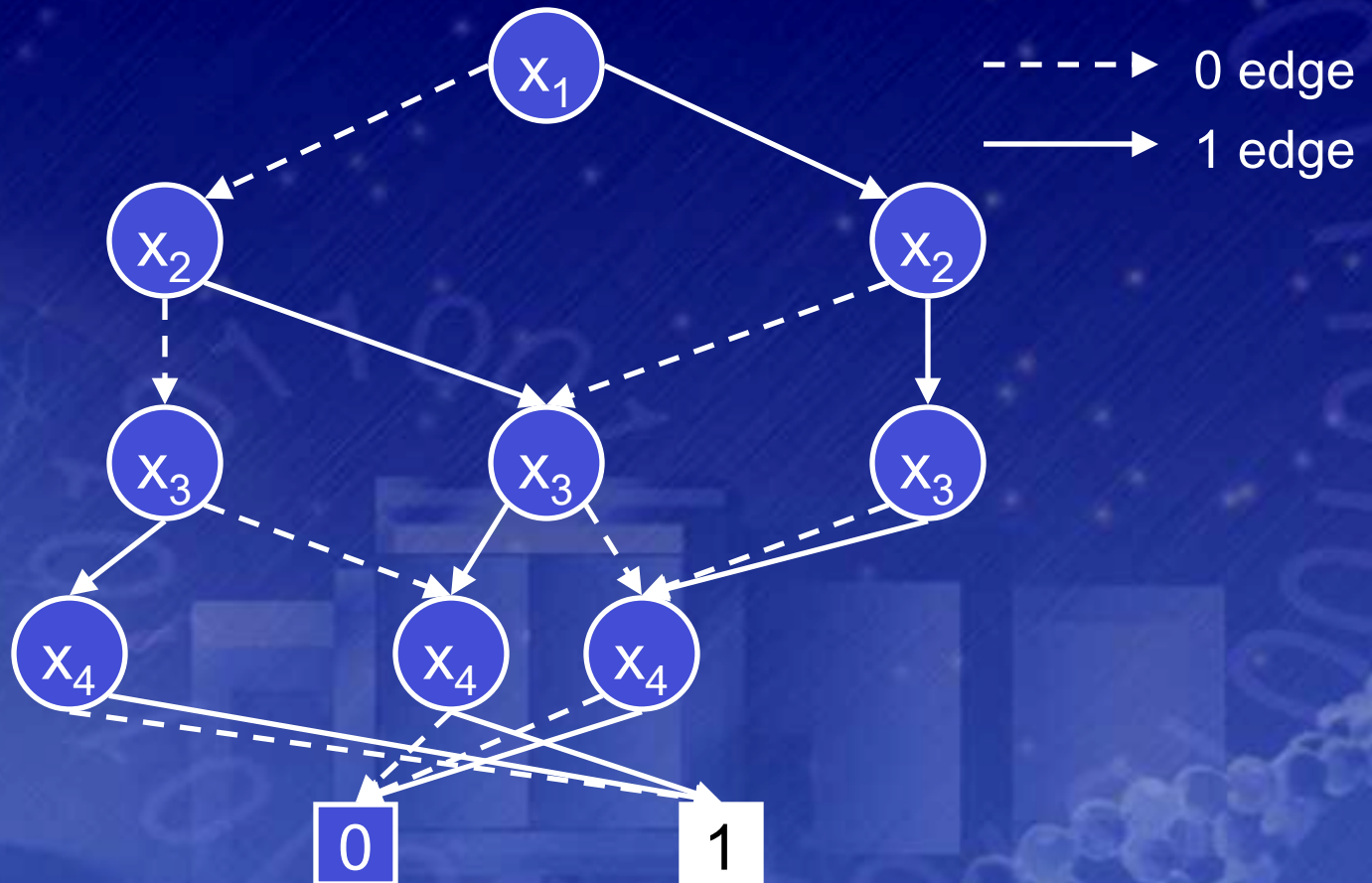
Binary Decision Diagrams

- Collapse redundant nodes.



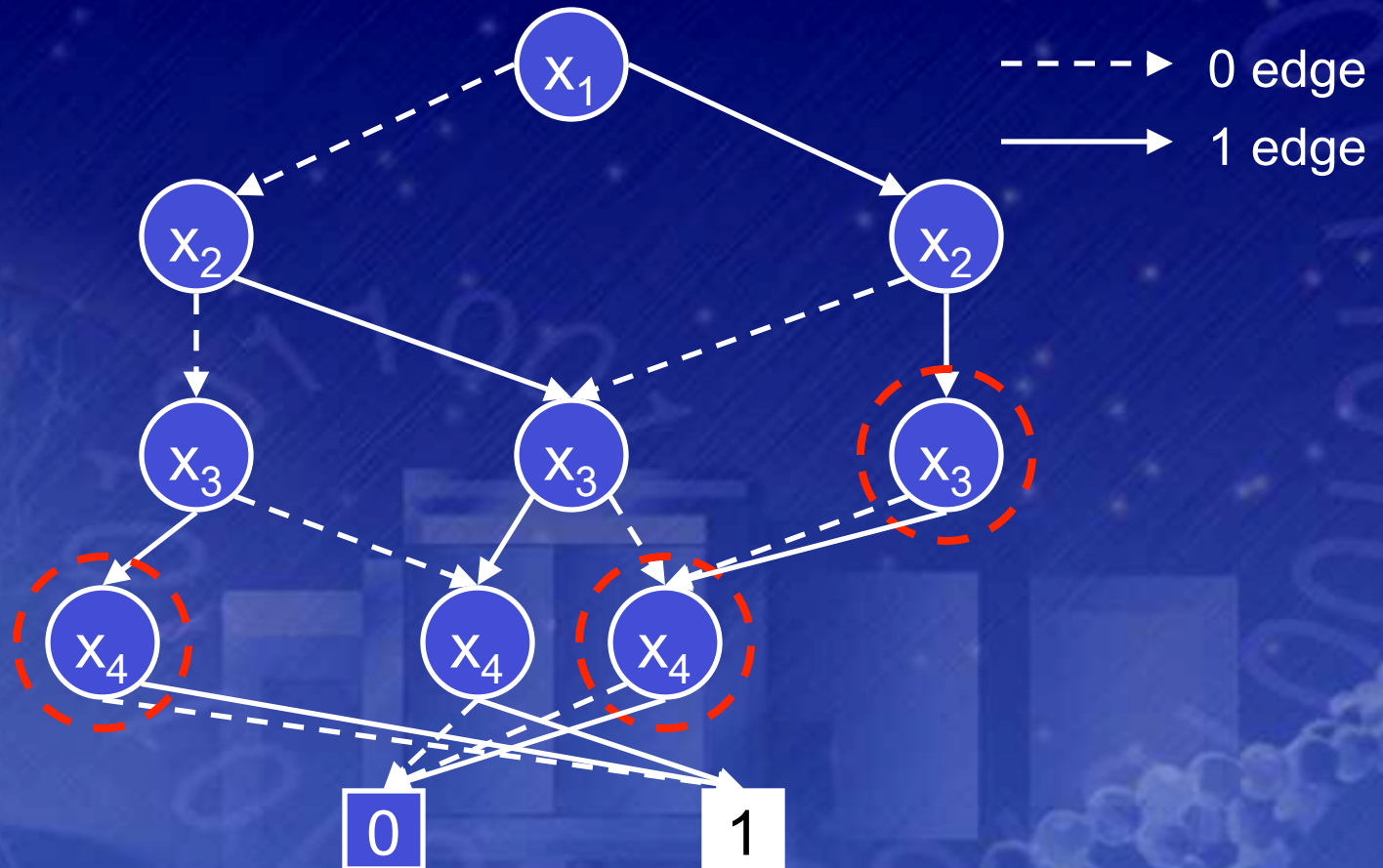
Binary Decision Diagrams

- Collapse redundant nodes.



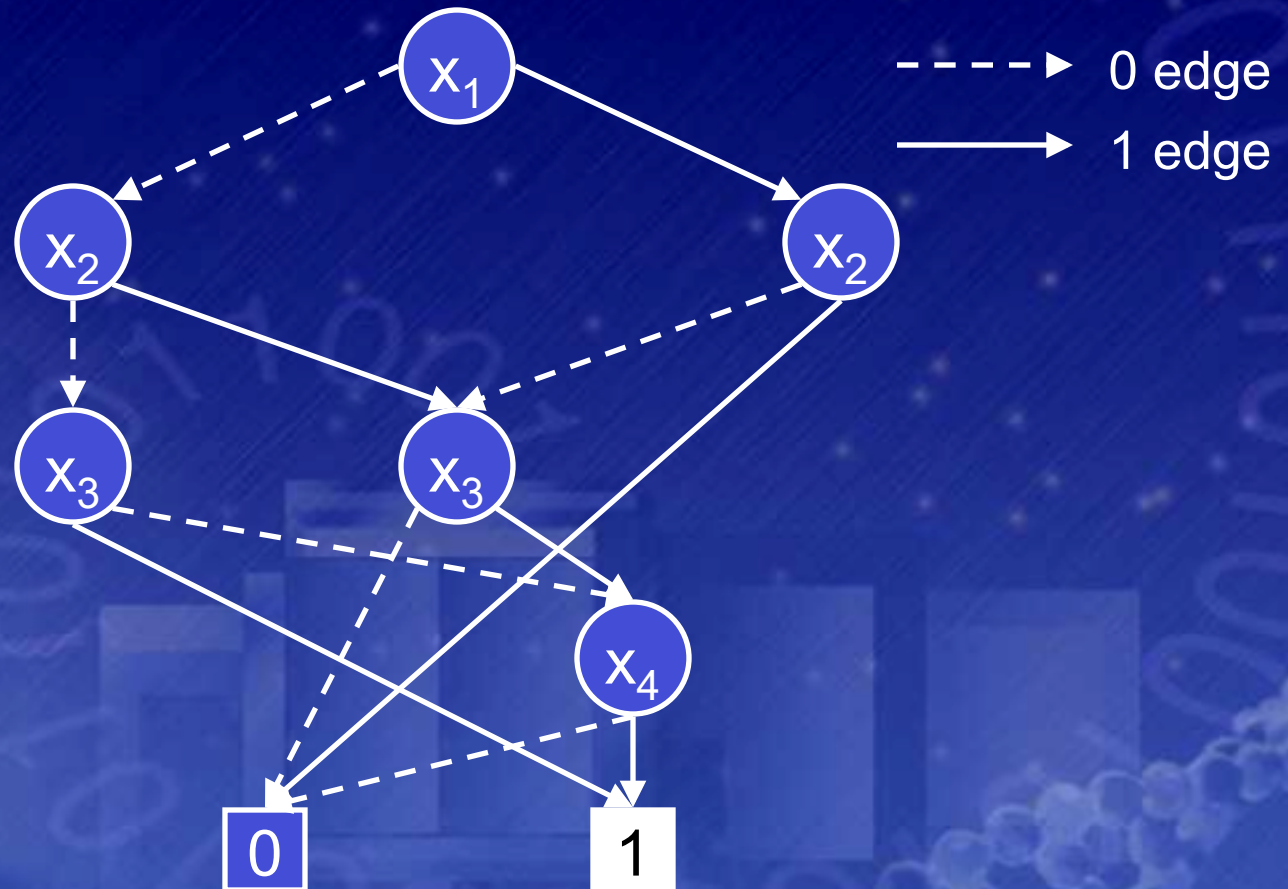
Binary Decision Diagrams

- Eliminate unnecessary nodes.



Binary Decision Diagrams

- Eliminate unnecessary nodes.

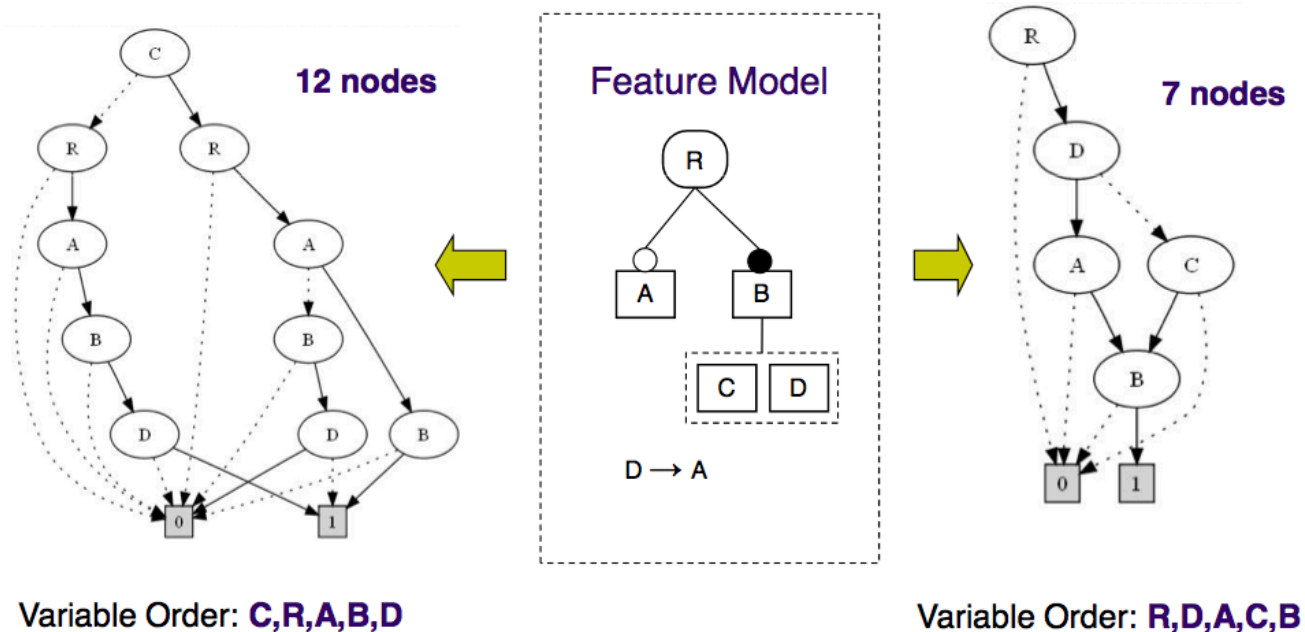


Binary Decision Diagrams (BDDs)

- Very efficient structure for most of the satisfiability operations
- Polynomial in time for checking satisfiability and determining equivalence between two BDDs
- Graph traversal
- So great?

Binary Decision Diagrams (BDDs): Theoretical Problem

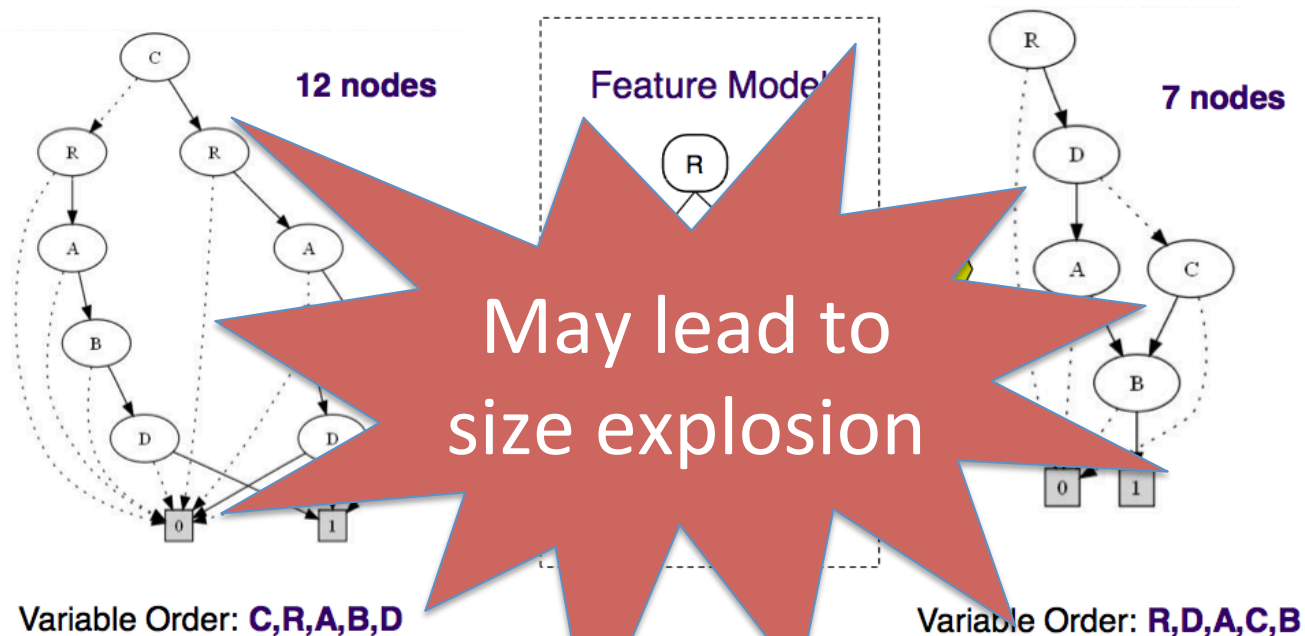
- The size of the BDD is very sensitive to the order of the BDD variables
 - e.g. two equivalent BDDs for the same feature



[Mendonca, slide]

Binary Decision Diagrams (BDDs): Theoretical Problem

- The size of the BDD is very sensitive to the order of the BDD variables
 - e.g. two equivalent BDDs for the same feature



[Mendonca, slide]

Binary Decision Diagrams (BDDs): Theoretical Problem

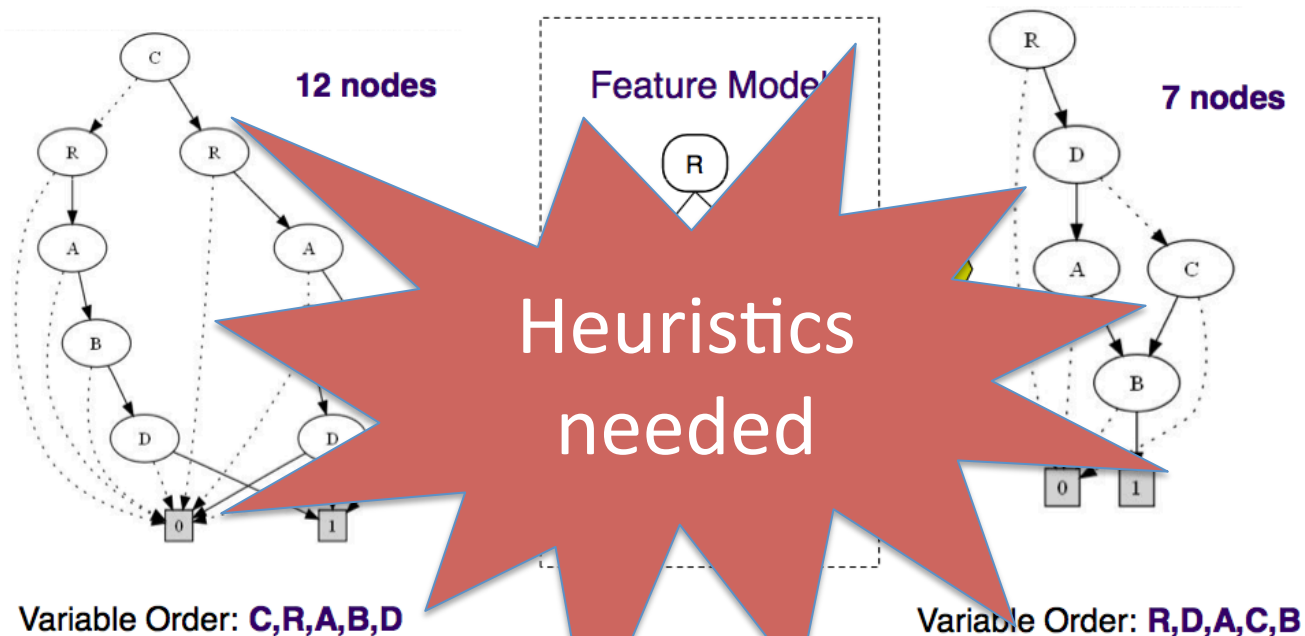
- The size of the BDD is very sensitive to the order of the BDD variables
 - e.g. two equivalent BDDs for the same feature



[Mendonca, slide]

Binary Decision Diagrams (BDDs): Practical Problem

- The size of the BDD is very sensitive to the order of the BDD variables. In practice: **BDDs cannot be build for feature models with 2000+ features**



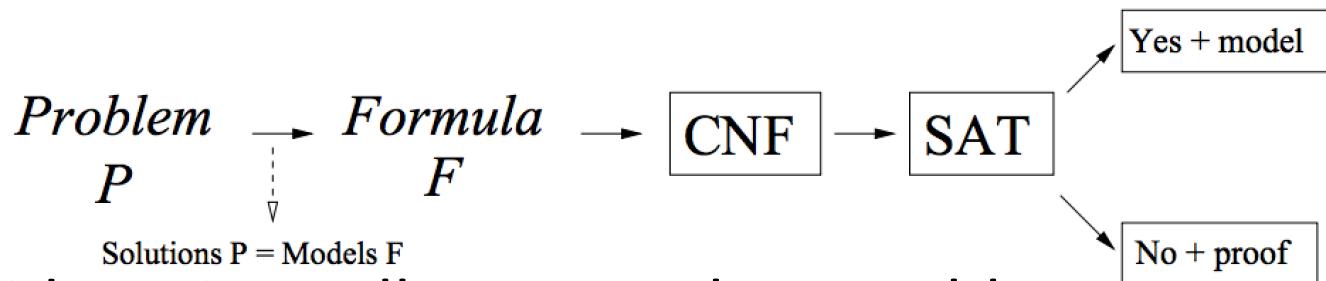
[Mendonca, slide]

How to automate analysis
of your feature models?

Let us try with SAT solvers

Satisfiability (SAT) solver

- A “SAT solver” is a program that automatically decides whether a propositional logic formula is satisfiable.
 - If it is satisfiable, a SAT solver will produce an example of a truth assignment that satisfies the formula.



- Basic idea: since all NP-complete problems are mutually reducible:
 - Write one really good solver for NP-complete problems (in fact, get lots of people to do it. Hold competitions.)
 - Translate your NP-complete problems to that problem.

SAT solver and CNF

- All current fast SAT solvers work on CNF
- Terminology:
 - A literal is a propositional variable or its negation (e.g., p or $\neg q$).
 - A clause is a disjunction of literals (e.g., $(p \vee \neg q \vee r)$). Since \vee is associative, we can represent clauses as lists of literals.
- A formula is in conjunctive normal form (CNF) if it is a conjunction of clauses
 - e.g., $(p \vee q \vee \neg r) \wedge (\neg p \vee s \vee t \vee \neg u)$

SAT solver and Unit Propagation

- Whenever all the literals in a clause are false except one, the remaining literal must be true in any satisfying assignment (such a clause is called a **unit clause**).
 - Therefore, the algorithm can assign it to true immediately. After choosing a variable there are often many unit clauses.
 - Setting a literal in a unit clause often creates other unit clauses, leading to a cascade.

$$\{\neg p \vee q, \neg p \vee \neg q \vee r, p, \neg r\}.$$

$\neg p \vee q$	q
$\neg p \vee \neg q \vee r$	$\neg q \vee r$
p	
$\neg r$	$\neg r$

- A good SAT solver often spends 80-90% of its time in unit propagation.

$$\begin{aligned}\mathcal{F}_{\text{unit}} := & (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \\ & (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \\ & (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{\text{unit}} := & (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \\ & (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \\ & (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)\end{aligned}$$

$$\varphi = \{x_1=1\}$$

$$\begin{aligned}\mathcal{F}_{\text{unit}} := & (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \\ & (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \\ & (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)\end{aligned}$$

$$\varphi = \{x_1=1, x_2=1\}$$

$$\begin{aligned}
\mathcal{F}_{\text{unit}} := & (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \\
& (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \\
& (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)
\end{aligned}$$

$$\varphi = \{x_1=1, x_2=1, x_3=1\}$$

$$\begin{aligned}\mathcal{F}_{\text{unit}} := & (\neg x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \\ & (\neg x_1 \vee x_2) \wedge (x_1 \vee x_3 \vee x_6) \wedge (\neg x_1 \vee x_4 \vee \neg x_5) \\ & (x_1 \vee \neg x_6) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_5 \vee \neg x_6)\end{aligned}$$

$$\varphi = \{x_1=1, x_2=1, x_3=1, x_4=1\}$$

SAT solver and Unit Propagation

BCP():

Repeatedly search for unit clauses, and
set unassigned literal to required value.

If a literal is assigned conflicting values, return F
else return T;

satisfy(ϕ) {

if every clause of ϕ has a true literal, return T;

if BCP() == F, return F;

assign appropriate values to all pure literals;

choose an $x \in V$ that is unassigned in A ,

and choose $v \in \{T, F\}$.

$A(x) = v$;

if satisfy(ϕ) return T;

$A(x) = \neg v$;

if satisfy(ϕ) return T;

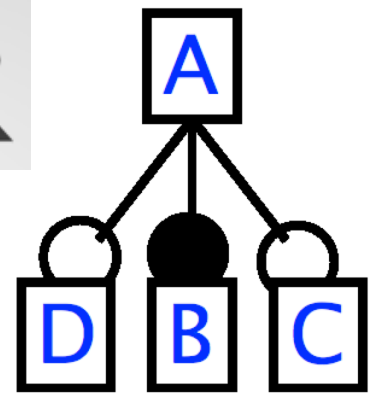
unassign $A(x)$; // undo assignment for backtracking.

return F; }

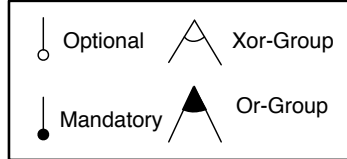
How to automate analysis of your feature models?

Let us use BDDs and SAT solvers

$A \wedge$
 $A \Leftrightarrow B \wedge$
 $C \Rightarrow A \wedge$
 $D \Rightarrow A$



FM



```

fm1bis = FM ("foo3.dimacs")
fm1bisbis = FM ("foo3.constraints")

```

```

fm1> fm1 = FM ("output/fm1.tvl")
root A {
  group [ 3..3 ] {
    opt D {
    },
    B {
    },
    opt C {
    }
  }
}
fm1: (FEATURE_MODEL) A: [D] B [C] ;

```

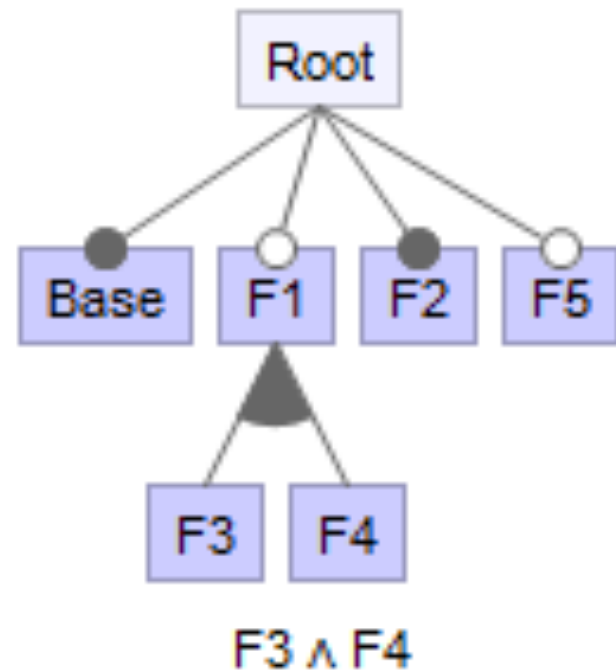
```

fm1> c1 = cores fm1
fm1> s1 c1: (SET) {B;A}
s1: (SET) {B;A}
fm1> c1bis = cores fm1bis
c1bis: (SET) {B;A}
fm1> compare fm1 fm1bis
res7: (STRING) REFACTORING {B;A;B;D}}
fm1> compare fm1bis fm1bisbis
res8: (STRING) REFACTORING {B;A};{B;A}}
fm1> s1 res3: (BOOLEAN) true
fm1> s1 res6: (BOOLEAN) true
res4: (BOOLEAN) true

```

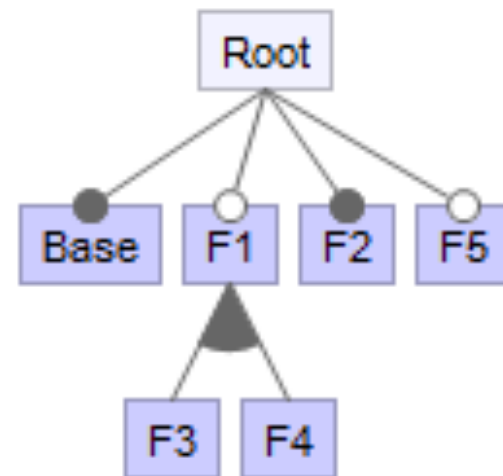
Consistency

- SAT-Solver
 - SAT(FM)



Core and dead features

- Dead : $SAT(FM \wedge F)$
- Core: $SAT(FM \wedge \text{not}(F))$



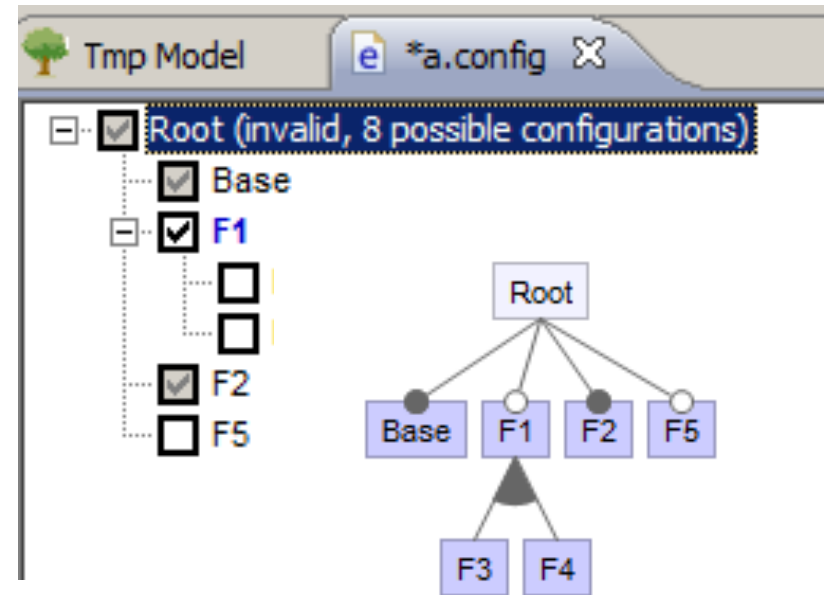
$F5 \Rightarrow F4 \vee \text{Base}$

$F3 \Rightarrow F2 \wedge F5$

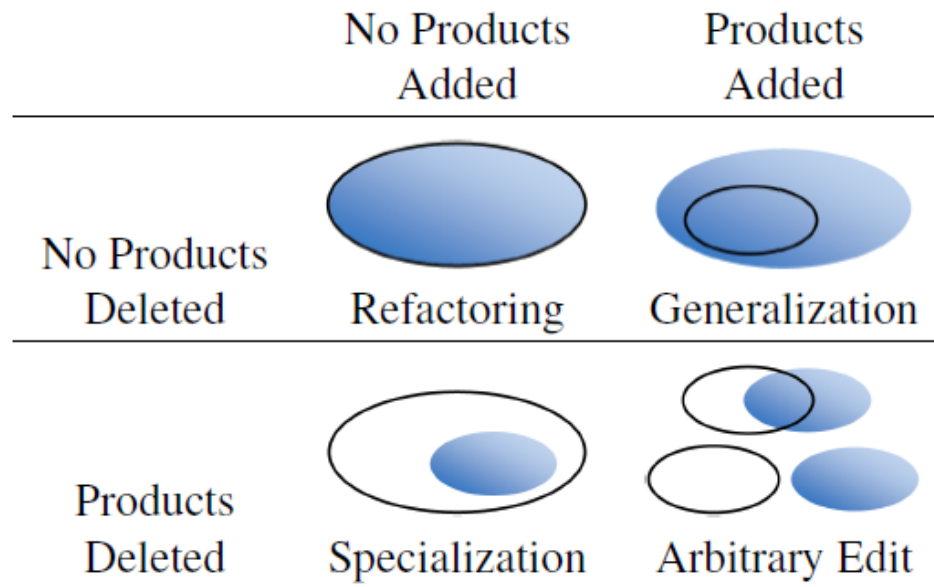
$\neg(F4 \wedge F2)$

Partial configuration

- $SAT(FM \wedge PK \wedge F)$
- $SAT(FM \wedge PK \wedge \text{not}(F))$



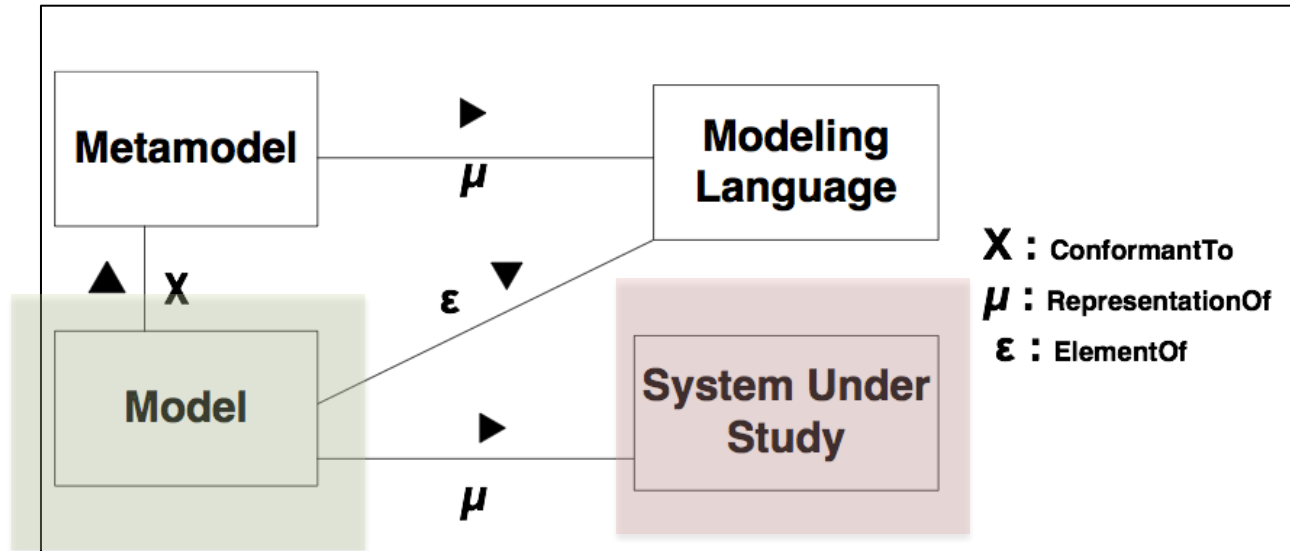
Relationship between feature models



- Refactoring
 - Tautology: $(FM1 \Leftrightarrow FM2)$
= not SAT(not $(FM1 \Leftrightarrow FM2)$)

Recap

Feature Models



▼ CarEquipment

▼ ● Healinging



AirConditioningFrontAndRear
 AirConditioning

▼ ● Comfort

○ AutomaticHeadLights

▼ ● DrivingAndSafety

○ FrontFogLights

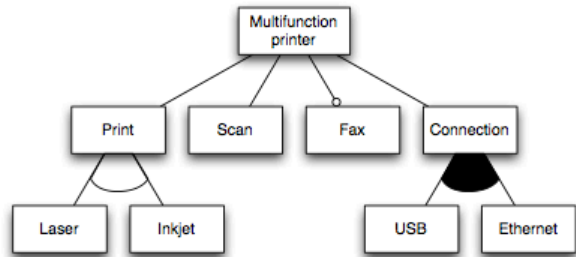
▼ Constraints

AutomaticHeadLights ⇒ FrontFogLights

R8 Spyder 5.2 FSI quattro R tronic
 Prix Total: 195.899,35 EUR
 Prix de base: 170.490,00 EUR
 Équipements optionnels: 25.409,35 EUR

Options list:
 - Chassis: 320,65 EUR
 - Freins: 931,70 EUR
 - Systèmes d'assistance: 1.373,35 EUR
 - Régulateur de vitesse: 1.790,80 EUR

Typical implementations



result



logics



solvers



Z3

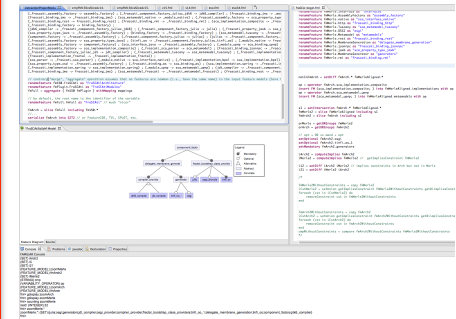
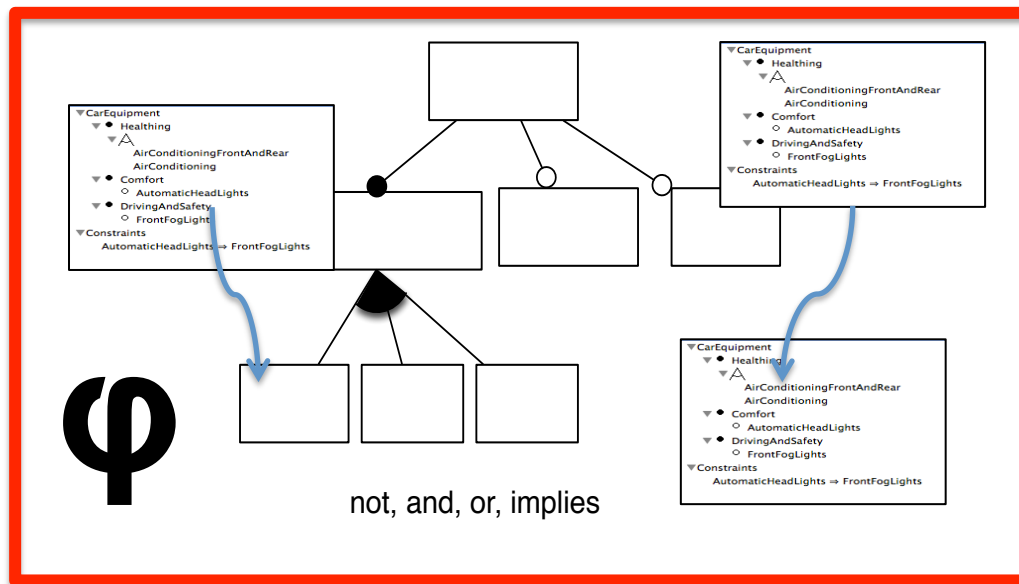
FAMiliAR

(FeAture Model script Language for manipulation and Automatic Reasoning)

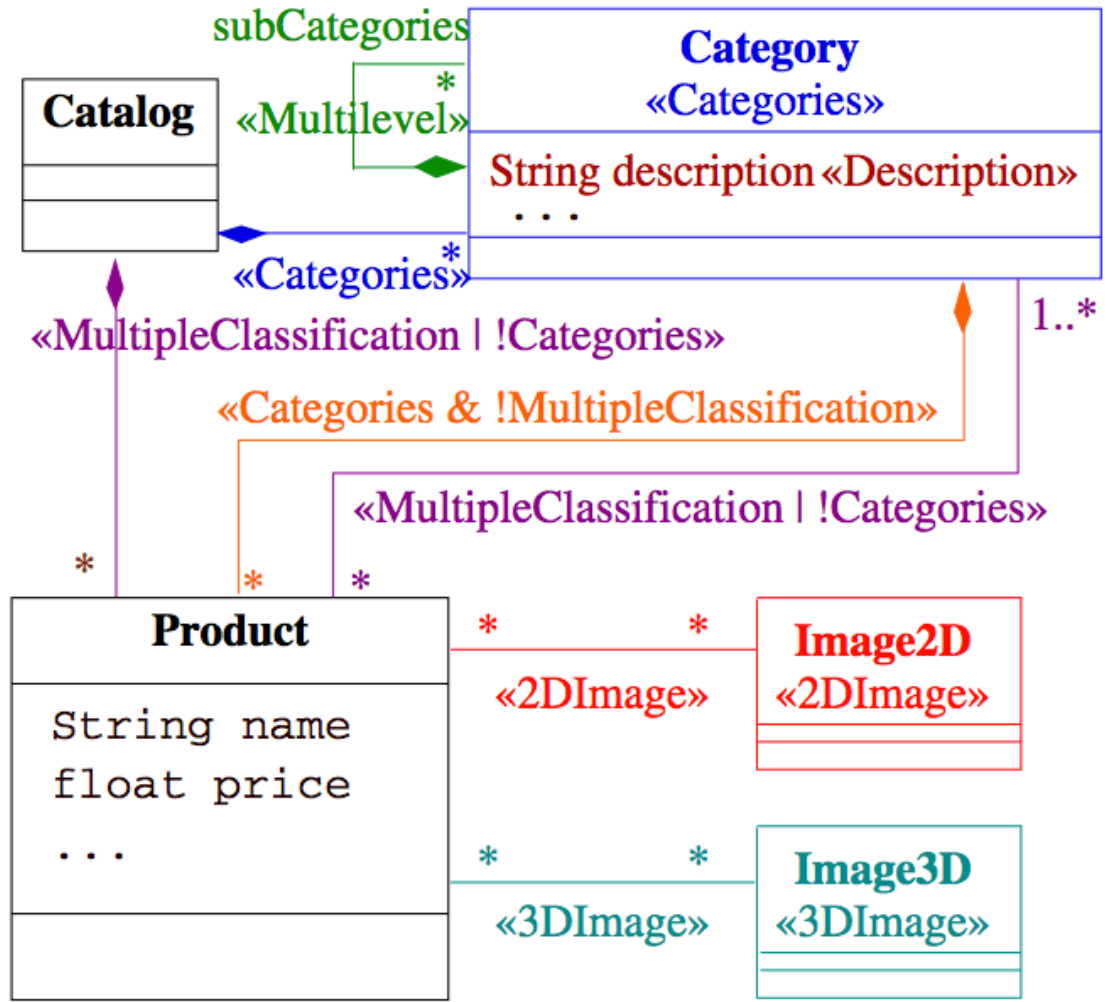
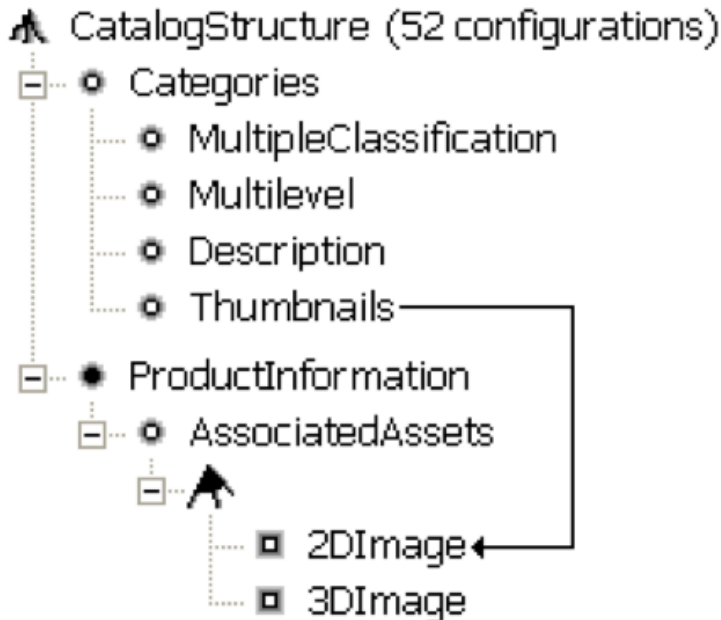
<http://familiar-project.github.com/>

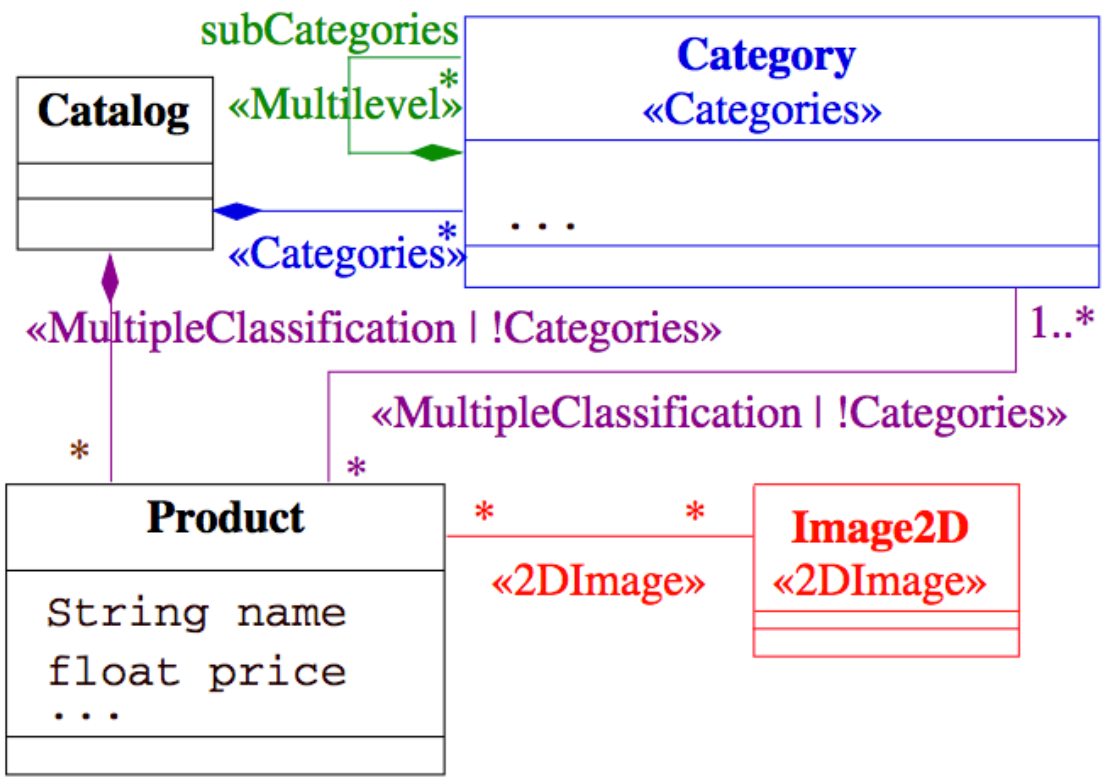
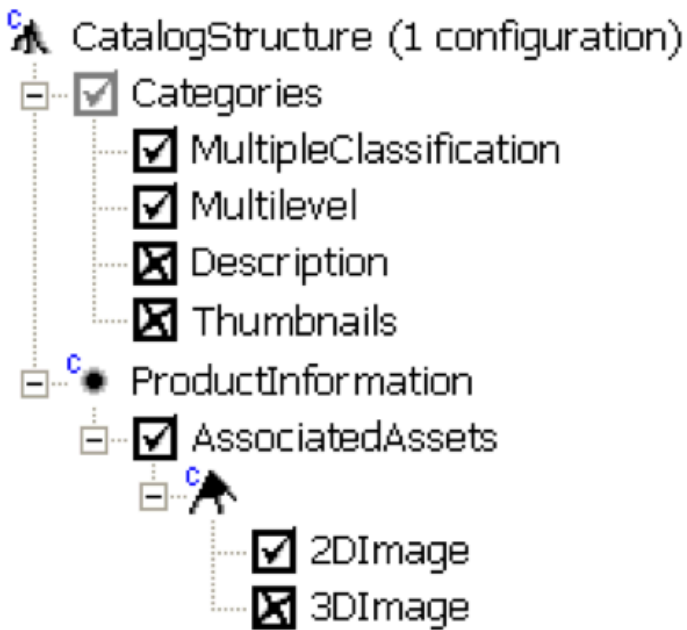


TVL
DIMACS

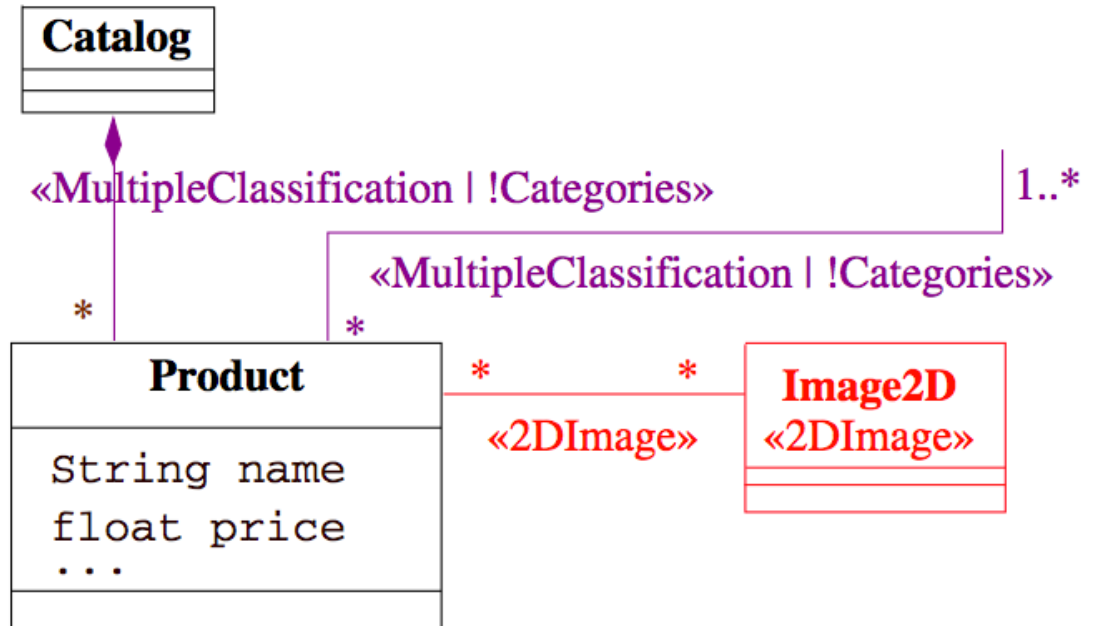
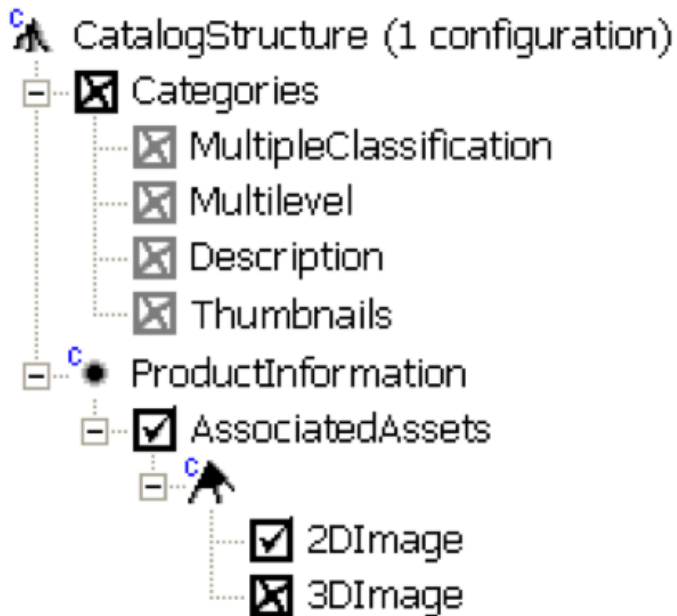


importing, exporting, composing, decomposing, editing, configuring, reverse engineering, computing "difs", refactoring, testing, and reasoning about (multiple) variability models

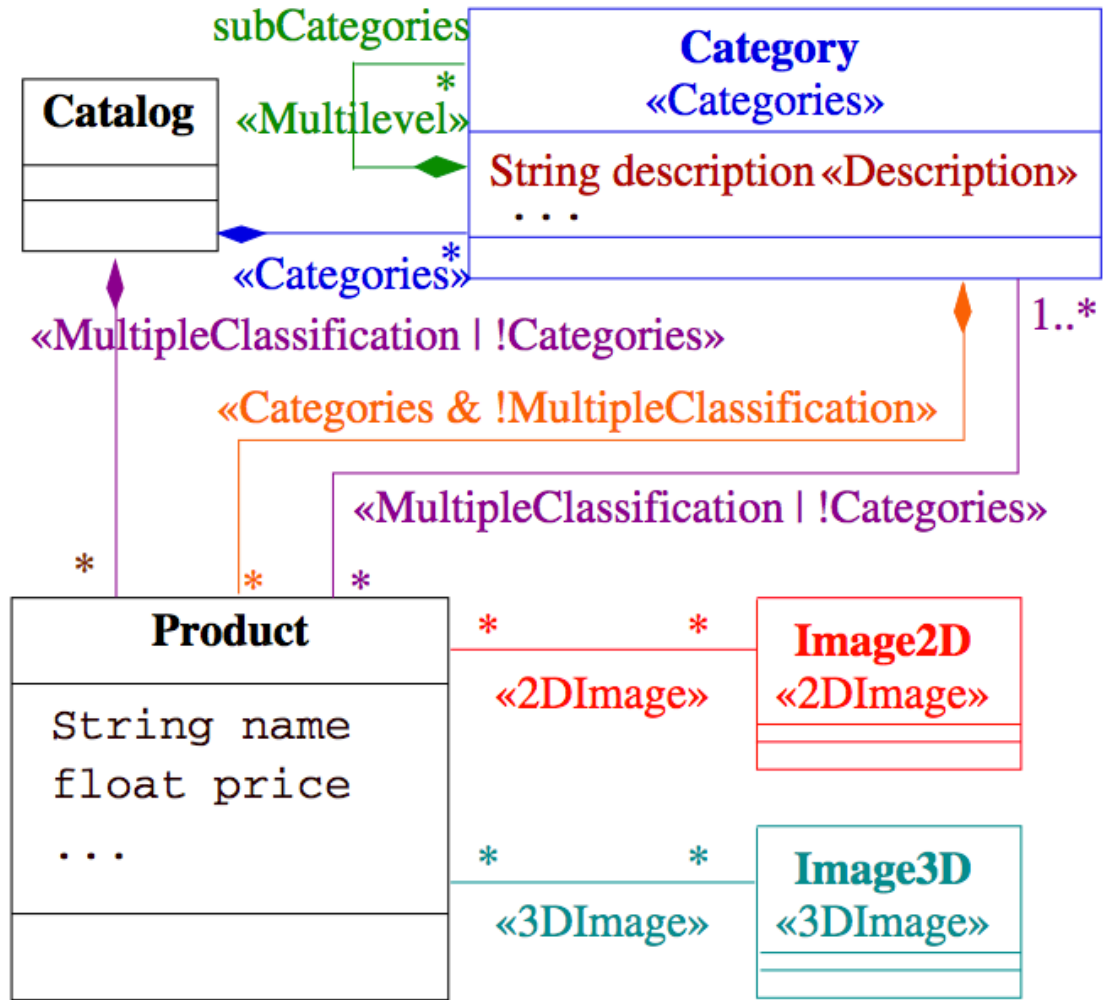
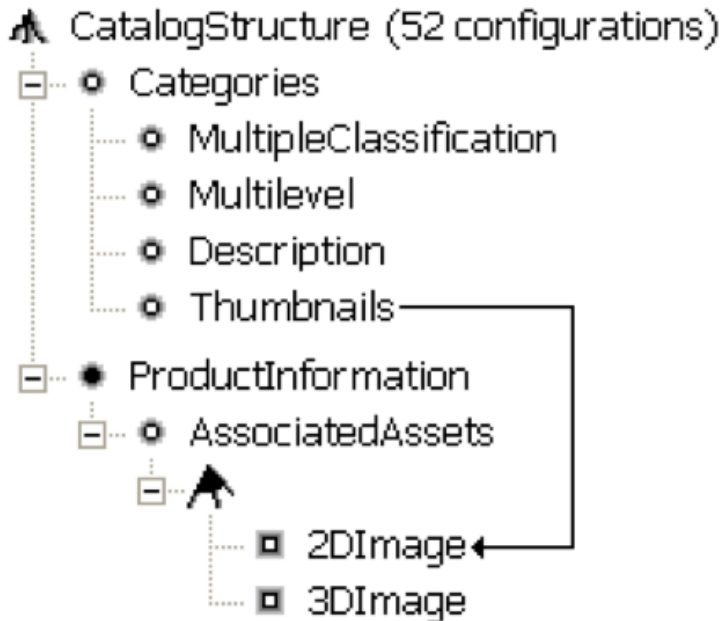




Ooops



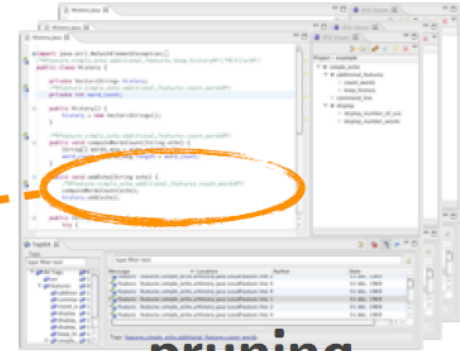
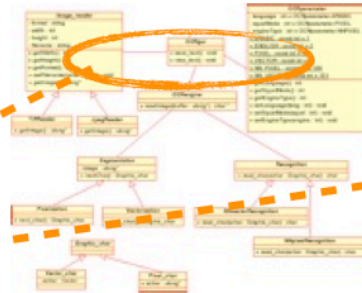
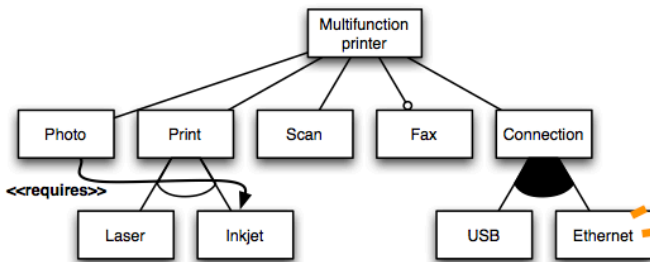
Safe composition? No!



Product Derivation

feature model

variable model and
code assets

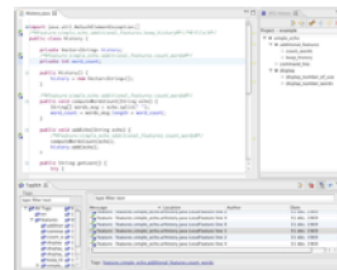


pruning,
composition,
weaving,
transformation
...

configuration

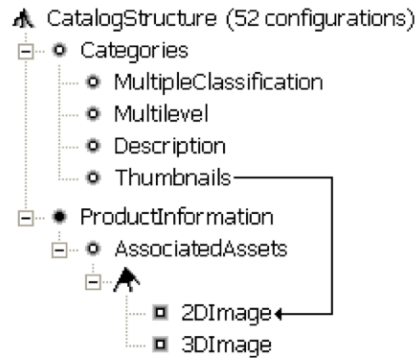
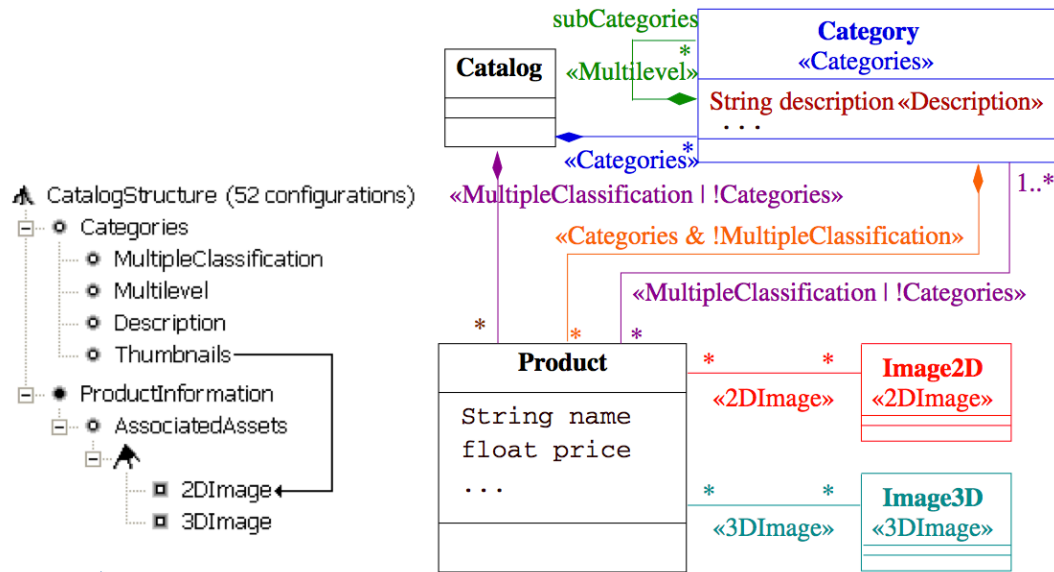
{ MP, Photo, Print, Inkjet, Scan,
Fax, Connection, USB, Ethernet }

product spec



product

Safe composition: how does it work?



qFM
 root:
 child-parent:

 group:
 mandatory:
 additional:

=
 $cs \wedge$
 $(ct \Rightarrow cs) \wedge (mc \Rightarrow ct) \wedge (ml \Rightarrow ct) \wedge$
 $(ds \Rightarrow ct) \wedge (tn \Rightarrow ct) \wedge (pi \Rightarrow cs) \wedge$
 $(aa \Rightarrow pi) \wedge (i2 \Rightarrow aa) \wedge (i3 \Rightarrow aa) \wedge$
 $(aa \Rightarrow \text{choice}_{1,2}(i2, i3)) \wedge$
 $(cs \Rightarrow pi) \wedge$
 $(tn \Rightarrow i2)$



**Another
 propositional
 formula**